Thursday Morning

Growth Modelling in Mplus

Using a set of repeated continuous measures of bodyweight

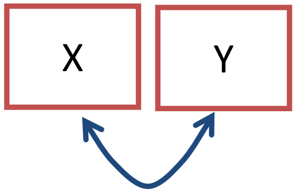
Growth modelling – Continuous Data

- Mplus model syntax refresher
- ALSPAC
- Confirmatory Factor Analysis (CFA)
- Latent Growth Modelling (LGM)
  - Path diagrams
  - Models of increasing complexity
  - Adding covariates
  - Extension to parallel processes
  - Practical

Mplus model syntax

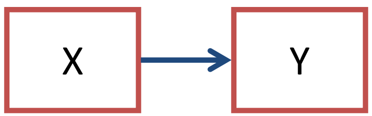
A quick refresher

Model statements: BY / WITH / ON



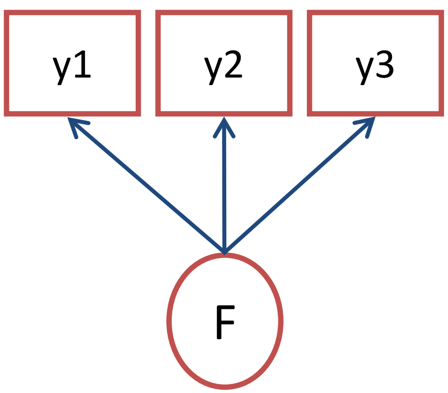
- X is correlated WITH Y

  X with Y;



- Y (outcome) is regressed on X (predictor)

  Y on X;



- F (the factor) is measured BY Y1 Y2 Y3

  F by Y1 Y2 Y3;

Variable means

- Stuff in a square bracket is a mean/intercept:

  [wt_7 wt_9 wt_11];

- It's just the same to say:

  [wt_7];

  [wt_9];

  [wt_11];

Variances

- No bracket, then it's a variance / residual variance:

  wt_7;

  wt_9;

  wt_11;

- Or

  wt_7 wt_9 wt_11;

Parameter equality constraints

- Three residual variances constrained to be equal:

```
wt_7      (1);
wt_9      (1);
wt_11     (1);
```

- Three intercept constrained to be equal:

```
[wt_7]    (2);
[wt_9]    (2);
[wt_11]   (2);
```

Parameter equality constraints

- Three residual variances constrained to be equal:

```
wt_7        (fixvar);
wt_9        (fixvar);
wt_11       (fixvar);
```

- Three intercept constrained to be equal:

```
[wt_7]      (fixmean);
[wt_9]      (fixmean);
[wt_11]     (fixmean);
```

Parameter equality constraints

- Three residual variances constrained to be equal:

  wt_7 (hamster);

  wt_9 (hamster);

  wt_11 (hamster);

- Three intercept constrained to be equal:

  [wt_7] (gerbil);

  [wt_9] (gerbil);

  [wt_11] (gerbil);

Fixing parameters

- Constraining a covariance to be zero:

```
X with Y@0;
```

- Constraining a mean to be zero:

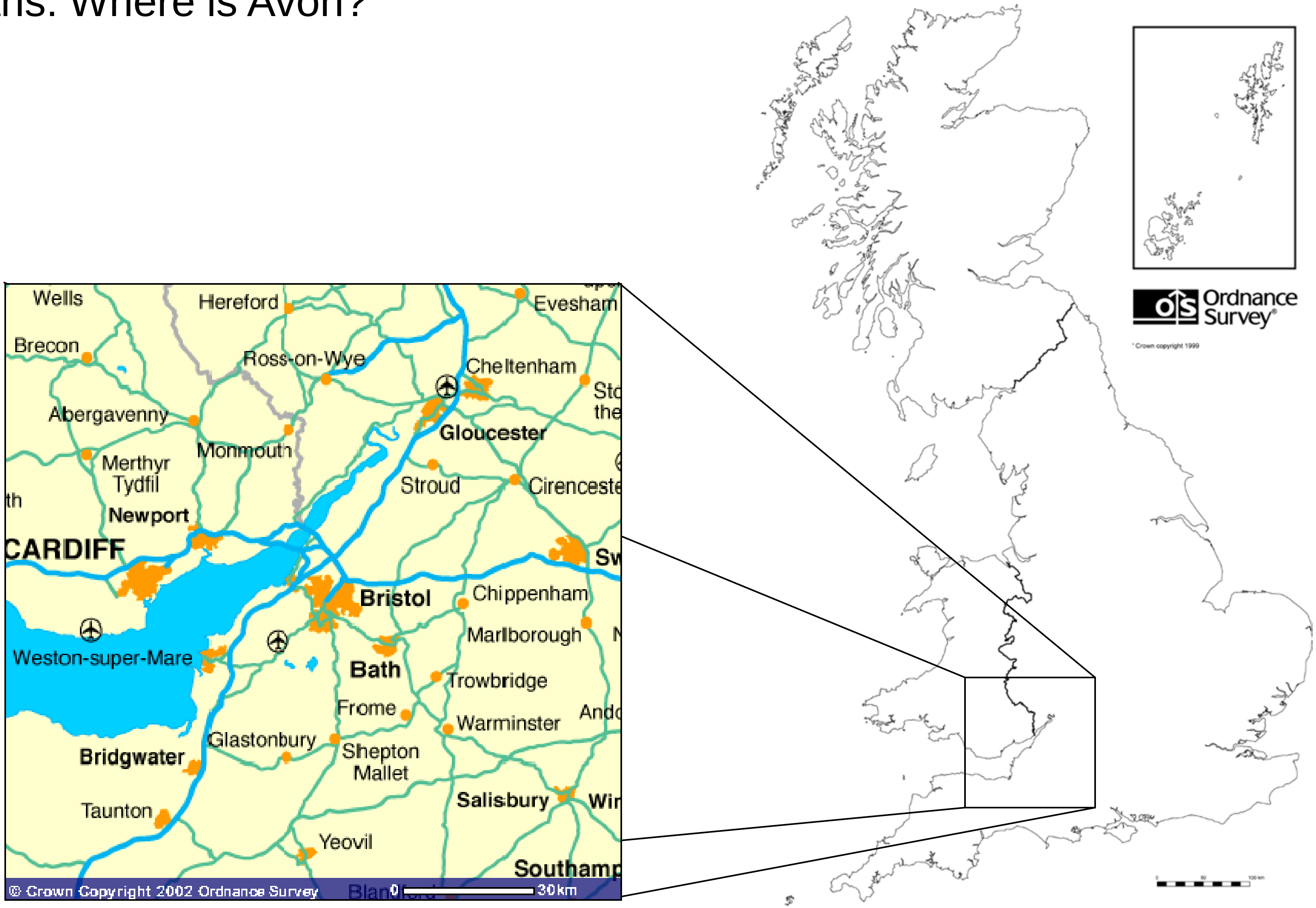
```
[wt_7@0];
```

- Constraining a variance to be zero:

```
i@0;
```

Where’s Avon to, my luvver?

trans: Where is Avon?



The old county of Avon

1. Known for it’s “ladies”
2. Had a very short name
3. Replaced in 1996 with
   - Bristol
   - North Somerset
   - Bath and North East Somerset
   - South Gloucestershire

   - Collectively known as “CUBA”

     (Counties which Used to Be Avon)

What is ALSPAC?

- "Avon Longitudinal Study of Parents and Children" AKA Children of the Nineties
- Cohort study of ~14,000 children and their parents, based in South-West England
- Eligibility criteria: Mothers had to be resident in Avon and have an expected date of delivery between April 1st 1991 and December 31st 1992

- Population-Based Prospective Birth-Cohort

What data does ALSPAC have?

- Self completion questionnaires
  - Mothers, Partners, Children, Teachers
- Hands on assessments
  - 10% sample tested regularly since birth
  - Yearly clinics for all since age 7
- Data from external sources
  - SATS from LEA, Child Health database
- Biological samples
  - DNA / cell lines

Bodyweight example

- Bodyweight measurements from ALSPAC clinic
- Three time points: 7, 9, 11 years
- N = 3,883 with complete data

- Summary statistics:-

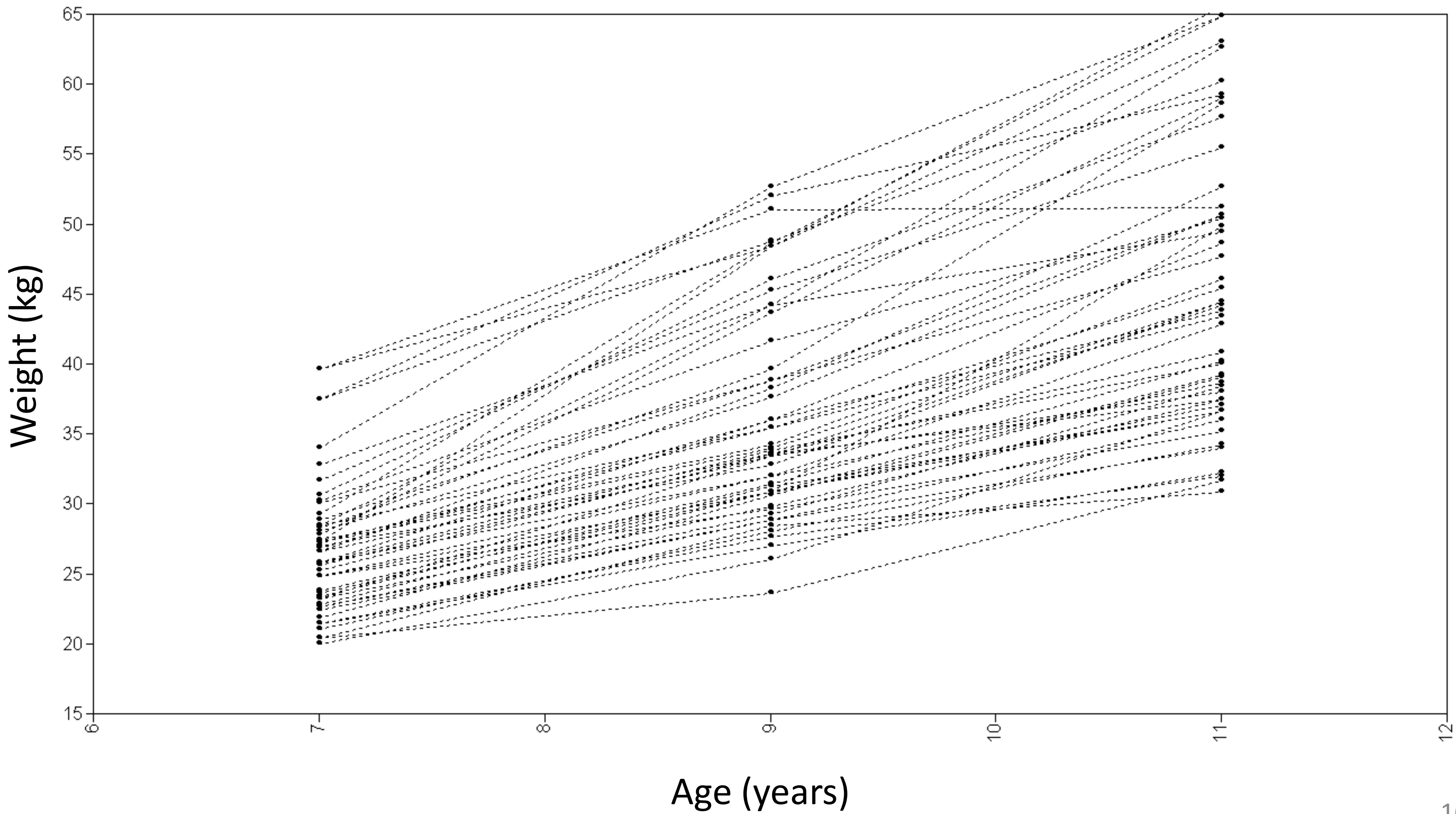
Means

WT7	WT9	WT11
25.532	34.219	43.214

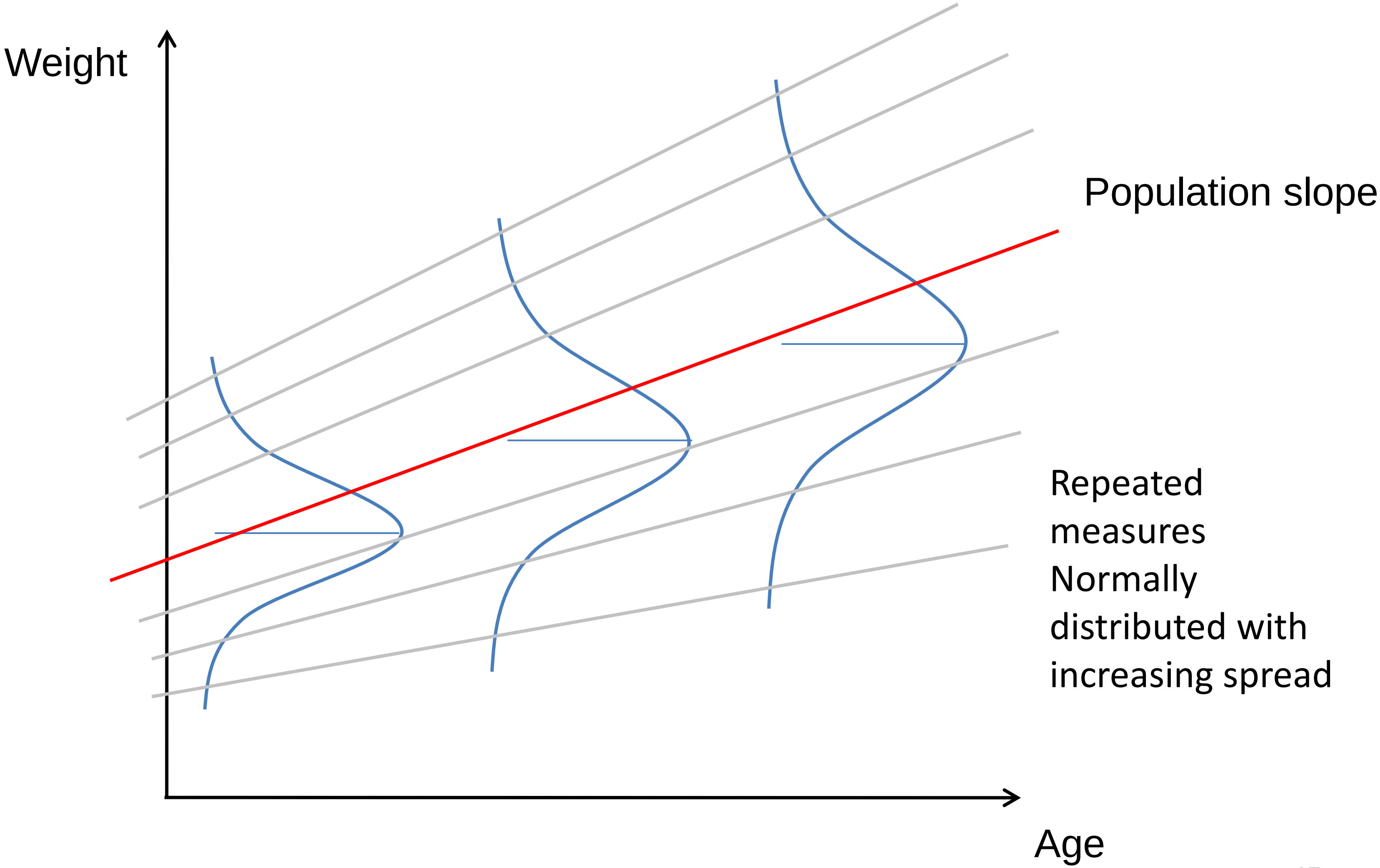
Covariances

	WT7	WT9	WT11
WT7	18.365		
WT9	27.543	49.787	
WT11	35.565	63.250	92.845

What the data looks like - v1

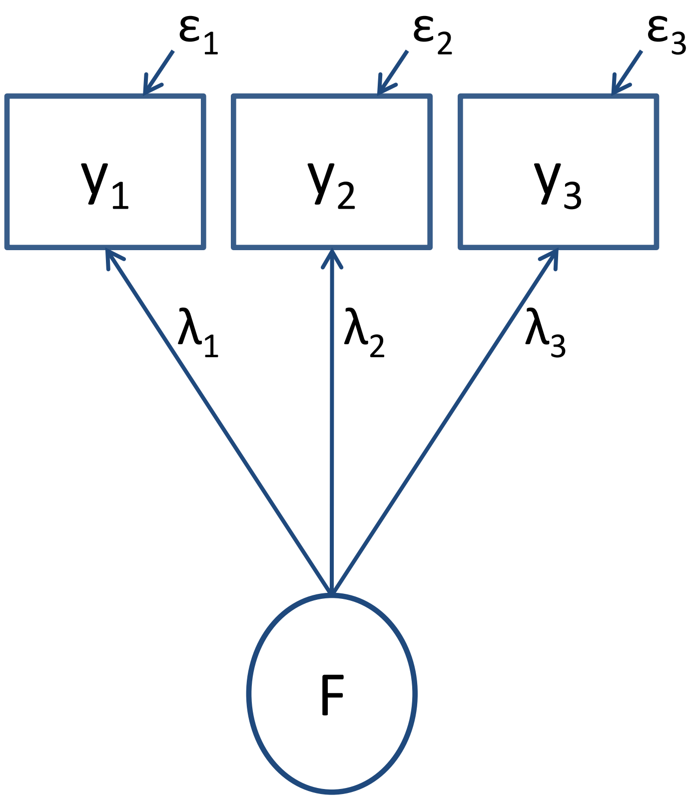


What the data looks like – v2



Repeated measures Normally distributed with increasing spread

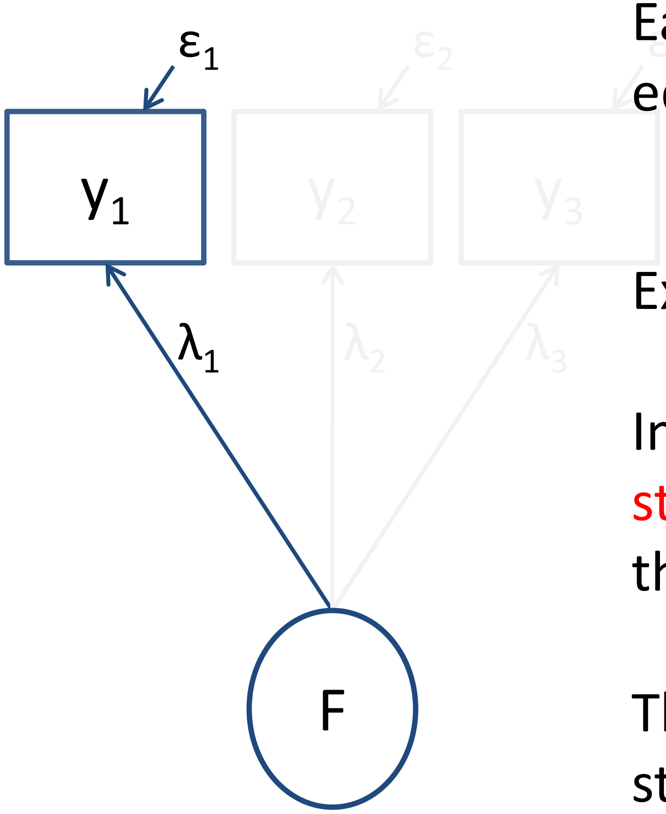
Confirmatory Factor Analysis



Of interest here are the loadings (λ_i), and the uniquenesses (ε_i) or residuals which have error variances σ_i^2

The interpretation of the factor is governed by the way it loads on the observed data.

Confirmatory Factor Analysis



Each component is a simple regression equation:

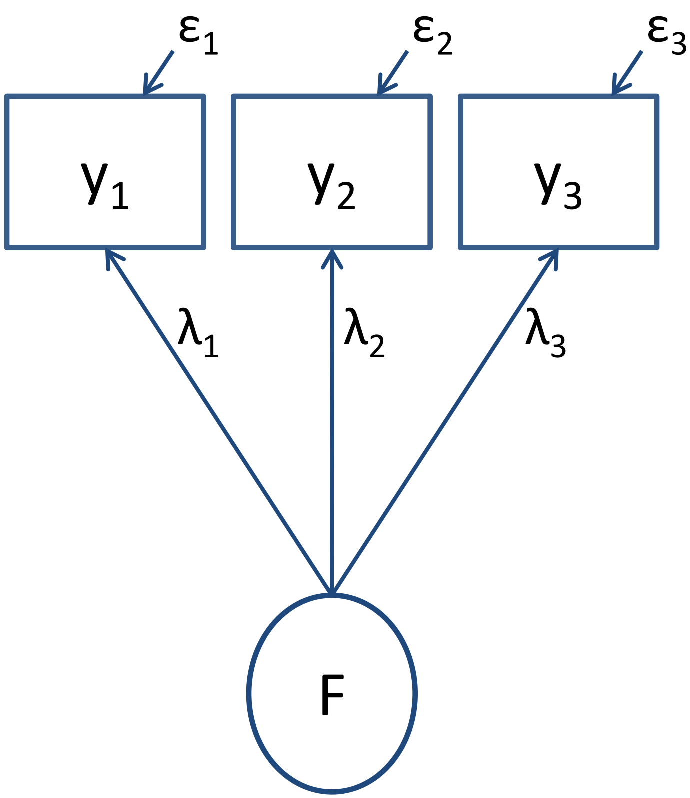
$$Y_1 = \lambda_1 F + \varepsilon_1$$

Except that F is *latent* rather than observed

In CFA we do not model the mean structure – hence there is no intercept in the above regression equation

Therefore CFA can be carried out using standardized measures

CFA - Covariance structure



The data to be modelled consists of a covariance matrix

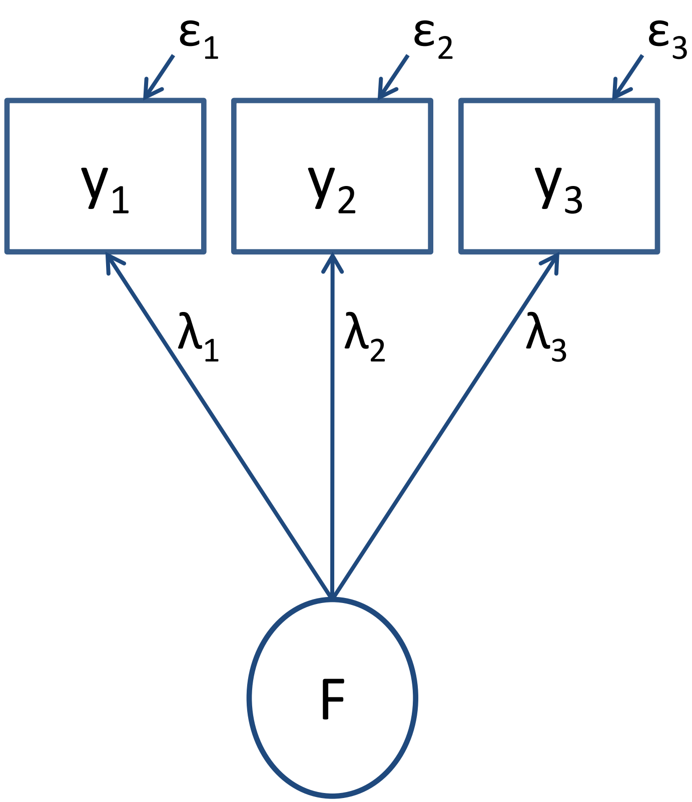
$$\begin{pmatrix} Var(y_1) & & \\ Cov(y_1,y_2) & Var(y_2) & \\ Cov(y_1,y_3) & Cov(y_1,y_3) & Var(y_3) \end{pmatrix}$$

We can write these six items in terms of the 7 parameters of interest:-

$$\lambda_1 \;\; \lambda_2 \;\; \lambda_3 \;\; \varepsilon_1 \;\; \varepsilon_2 \;\; \varepsilon_3 \;\; Var(F)$$

To many unknowns (under-identified) so we must apply a constraint.

CFA - in Mplus



Model:

```
F by y1* y2 y3;
F@1;
```

The factor F is measured by the set of items y1-y3

Here we have freed the estimation of the first loading and constrained the factor variance

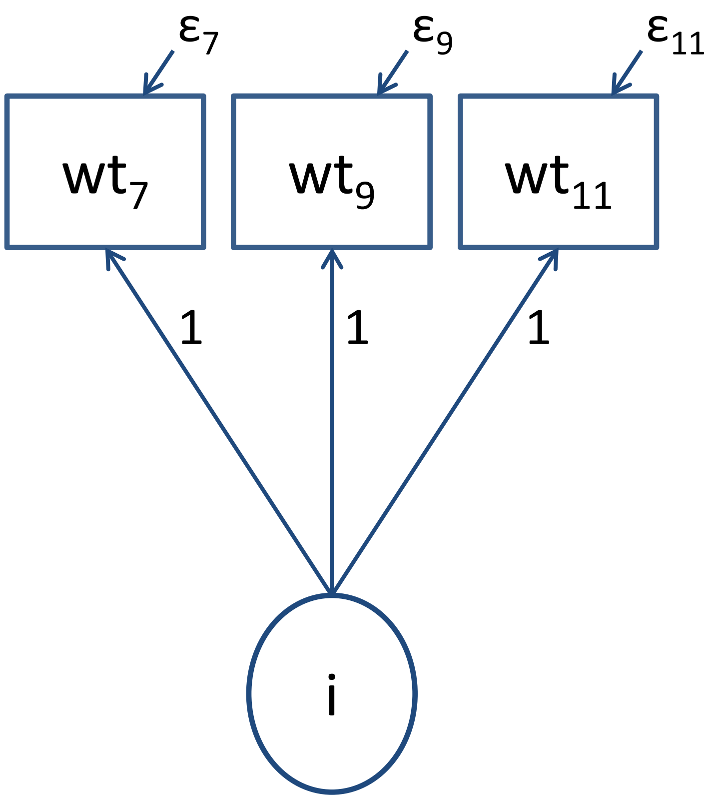
The Mplus default is to fix the first loading and estimate the variance of F. It’s just a matter of preference.

Now on to growth models

Not just for things that grow!

(Although things that don’t change at all can be boring to model)

A very simple Mplus growth model

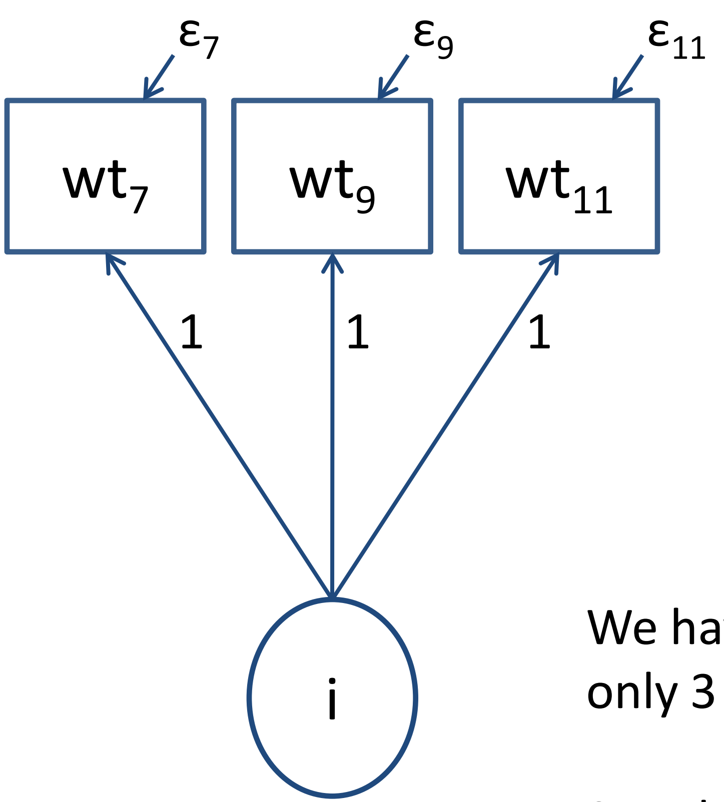


Similar figure to the CFA
However we have fixed the loadings

This model is saying that each repeated measure is a function of the intercept growth factor i with some random noise added

Mean(ε_i) = 0 hence we are assuming no growth, and i represents the constant level for each person

The Mean Structure



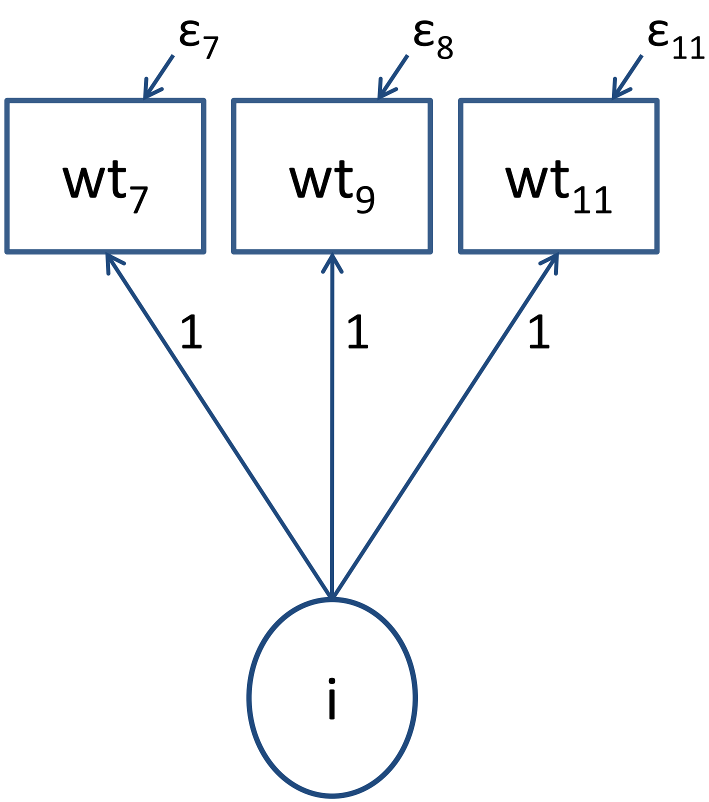
We can write the equations in the same way as for CFA but there is an additional term as the Y's are not standardized for these models:

$$wt_7 = \alpha_7 + i + \varepsilon_7$$
$$wt_9 = \alpha_9 + i + \varepsilon_9$$
$$wt_{11} = \alpha_{11} + i + \varepsilon_{11}$$

We have 4 things to estimate α_7, α_9, α_{11} & E(i) but only 3 measures: the means of wt_7 wt_9 wt_{11}

Standard convention to fix intercepts to zero and just estimate mean of the growth factor(s)

Degrees of Freedom



Covariance Structure

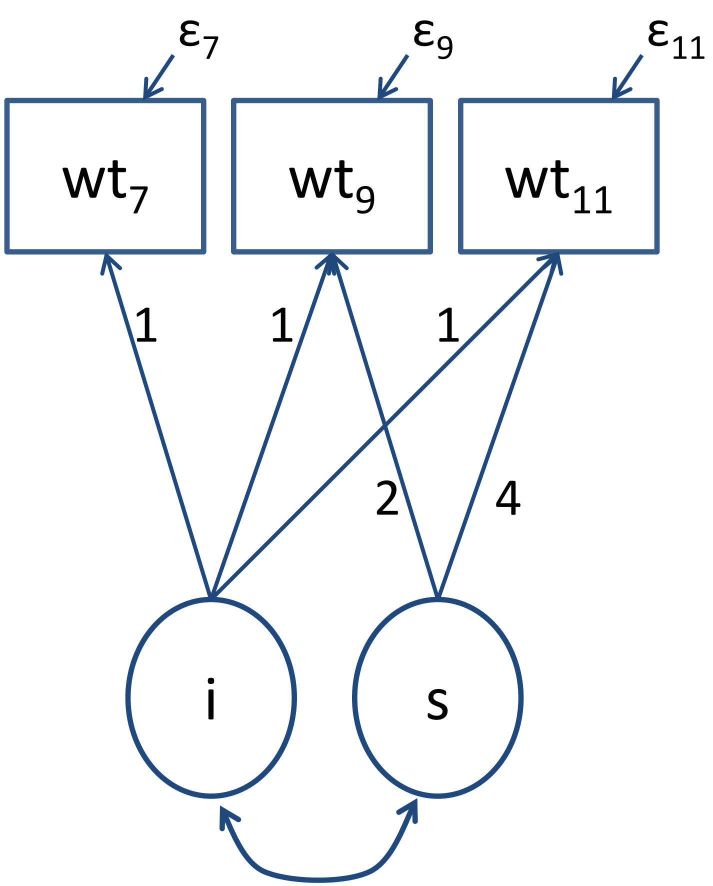
Covariance matrix of Y gives us 6 terms:
3 cov's and 3 vars.
Here we are estimating 4 things:
3 residual variances and the var(i)
Over-identified -> Good

Mean Structure

We have 3 terms – means of Y's
We are estimating 1 thing: mean(i)
Over-identified -> Good

You can't pool the d.f.
- estimate less in the mean structure to allow a more complex covariance structure

Linear Growth Model - Degrees of Freedom



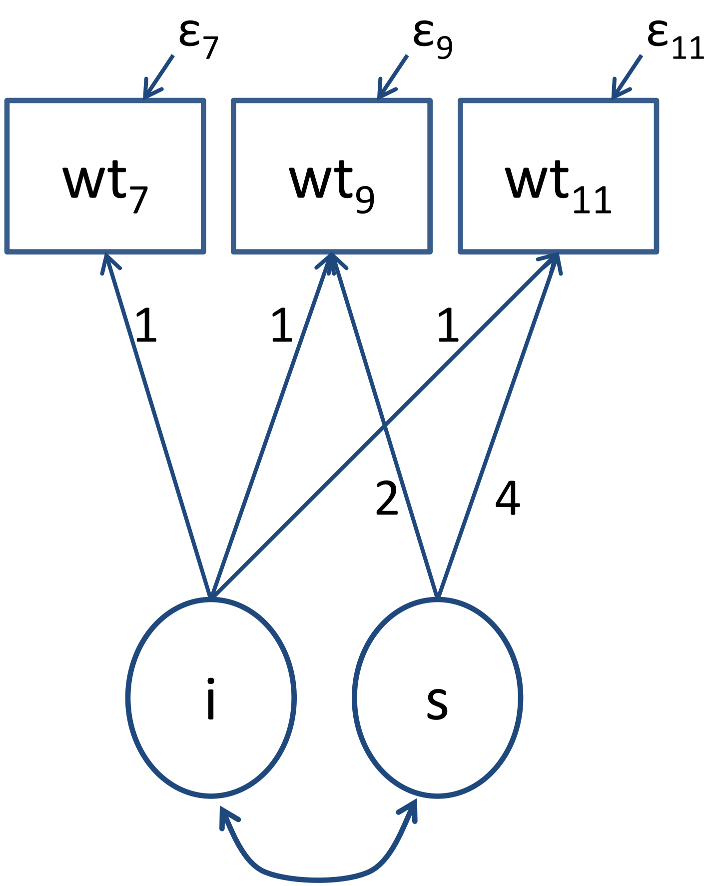
Covariance Structure

Covariance matrix of wt gives 6 terms:
3 cov's and 3 vars.
Here we are estimating 4 things:
A residual variance and the covariance matrix for the growth factors
Over-identified

Mean Structure

We have 3 terms – means of wt's
We are estimating 2 things: mean(i) and mean(s)
Over-identified

Linear Growth Model - Degrees of Freedom



Covariance Structure

What if we relaxed the residual variance constraint?

Var(i), Var(s), Cov(i,s), $(\sigma_7)^2$, $(\sigma_9)^2$, $(\sigma_{11})^2$

No d.f. spare -> Just identified

For 3 time points it is not possible to estimate more than 6 var/cov parameters.

Even if there are d.f. going spare in the mean structure model

Aim - Compare 5 models of bodyweight

- Fixed intercept / no slope
- Random intercept / no slope
- Fixed intercept / fixed slope
- Random intercept / fixed slope
- Random intercept / random slope

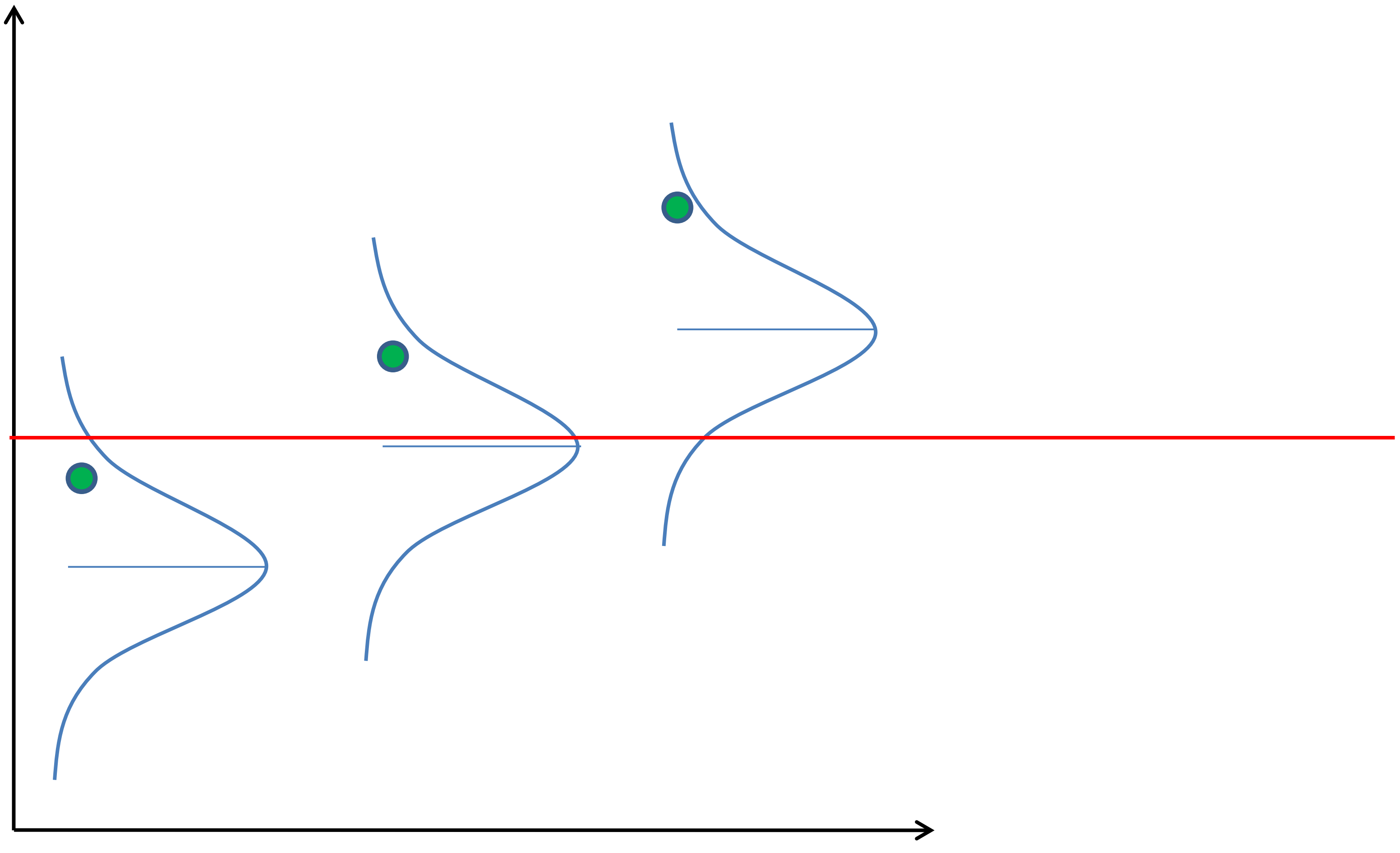
WHY?

- Statistical models based on assumptions
- Violated assumptions -> incorrect results

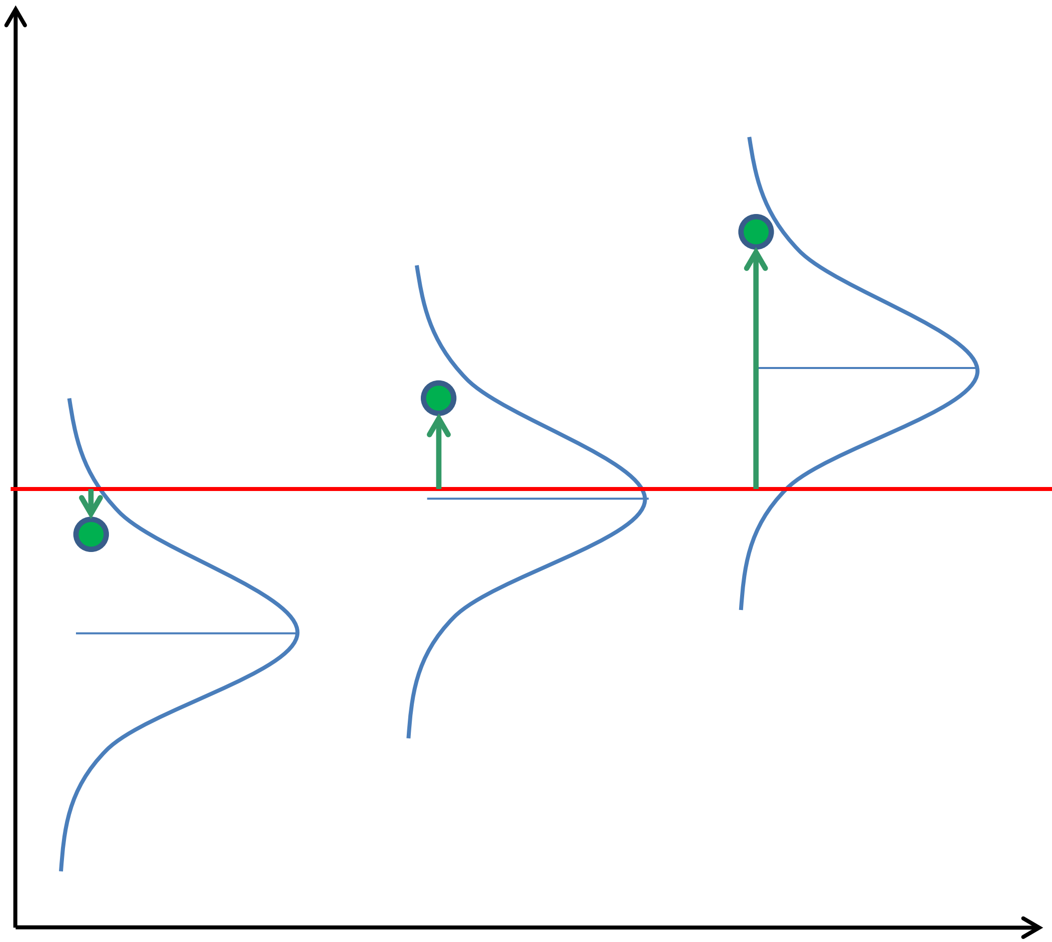
- Need to capture between and within person variability in growth as accurately as possible
- Leads to correct inferences when incorporating covariates

[1] Fixed intercept / no slope

Fixed intercept / no slope

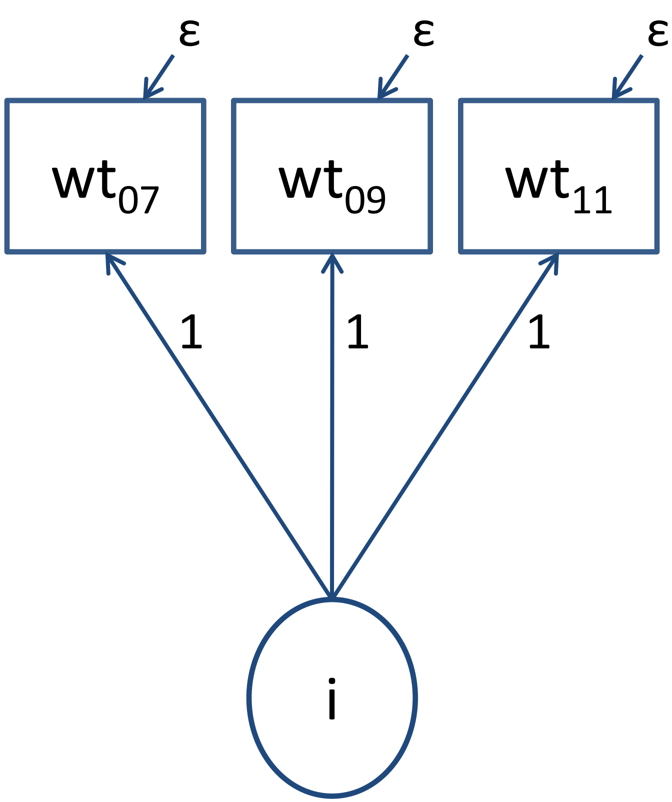


Fixed intercept / no slope



Assumes residuals (errors) have variance which is constant over time
+ adjacent residuals are uncorrelated

Fixed intercept / no slope



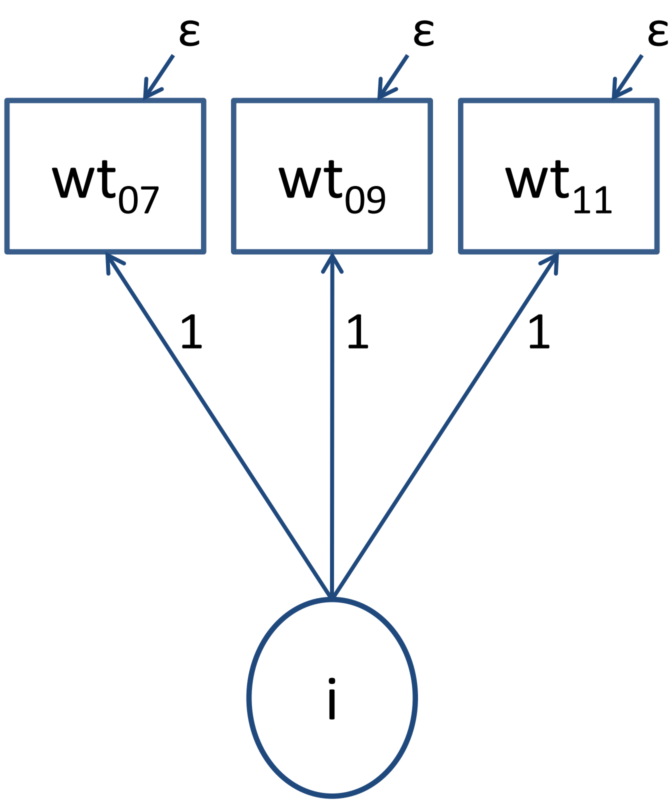
Single growth factor
Equal residual variances

```
model:
    i | wt_07@0 wt_09@2 wt_11@4;
    wt_07 (1);
    wt_09 (1);
    wt_11 (1);
    i@0;
```

Which is Mplus shorthand for:

```
    i by wt_07@1 wt_09@1 wt_11@1;
    [wt_07@0 wt_09@0 wt_11@0 i];
    wt_07 (1)
    wt_09 (1)
    wt_11 (1);
    i@0;
```

Fixed intercept / no slope



Single growth factor
Equal residual variances

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model:
    i | wt_07@0 wt_09@2 wt_11@4;
    wt_07 (1);
    wt_09 (1);
    wt_11 (1);
    i@0;
```

Which is the same as:

```
    i by wt_07@1 wt_09@1 wt_11@1;
    [wt_07@0 wt_09@0 wt_11@0 i];
    wt_07 (1)
    wt_09 (1)
    wt_11 (1);
    i@0;
```

Loadings are fixed

Fixed intercept / no slope - results

```
MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 I         |
    WT_07              1.000      0.000    999.000    999.000
    WT_09              1.000      0.000    999.000    999.000
    WT_11              1.000      0.000    999.000    999.000

 Means
    I                 34.322      0.095    360.167      0.000

 Intercepts
    WT_07              0.000      0.000    999.000    999.000
    WT_09              0.000      0.000    999.000    999.000
    WT_11              0.000      0.000    999.000    999.000

 Variances
    I                  0.000      0.000    999.000    999.000

 Residual Variances
    WT_07            105.782      1.386     76.319      0.000
    WT_09            105.782      1.386     76.319      0.000
    WT_11            105.782      1.386     76.319      0.000
```

Fixed intercept / no slope - results

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
I |				
WT_07	1.000	0.000	999.000	999.000
WT_09	1.000	0.000	999.000	999.000
WT_11	1.000	0.000	999.000	999.000
Means				
I	34.322	0.095	360.167	0.000
Intercepts				
WT_07	0.000	0.000	999.000	999.000
WT_09	0.000	0.		000
WT_11	0.000	0.		000
Variances				
I	0.000	0.		000
Residual Variances				
WT_07	105.782	1.386	76.319	0.000
WT_09	105.782	1.386	76.319	0.000
WT_11	105.782	1.386	76.319	0.000

(25.532 + 34.219 + 43.214)/3

All the variance in the dataset becomes residual variance (error) as nothing has been explained

Fixed intercept / no slope - residuals

Means

WT_07	WT_09	WT_11
25.532	34.219	43.214

Model Estimated Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
34.322	34.322	34.322

Residuals for Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
-8.790	-0.103	8.893

Fixed intercept / no slope - residuals

Covariances

	WT_07	WT_09	WT_11
WT_07	18.365		
WT_09	27.543	49.787	
WT_11	35.565	63.250	92.845

Model Estimated Covariances/Correlations/Residual Correlations

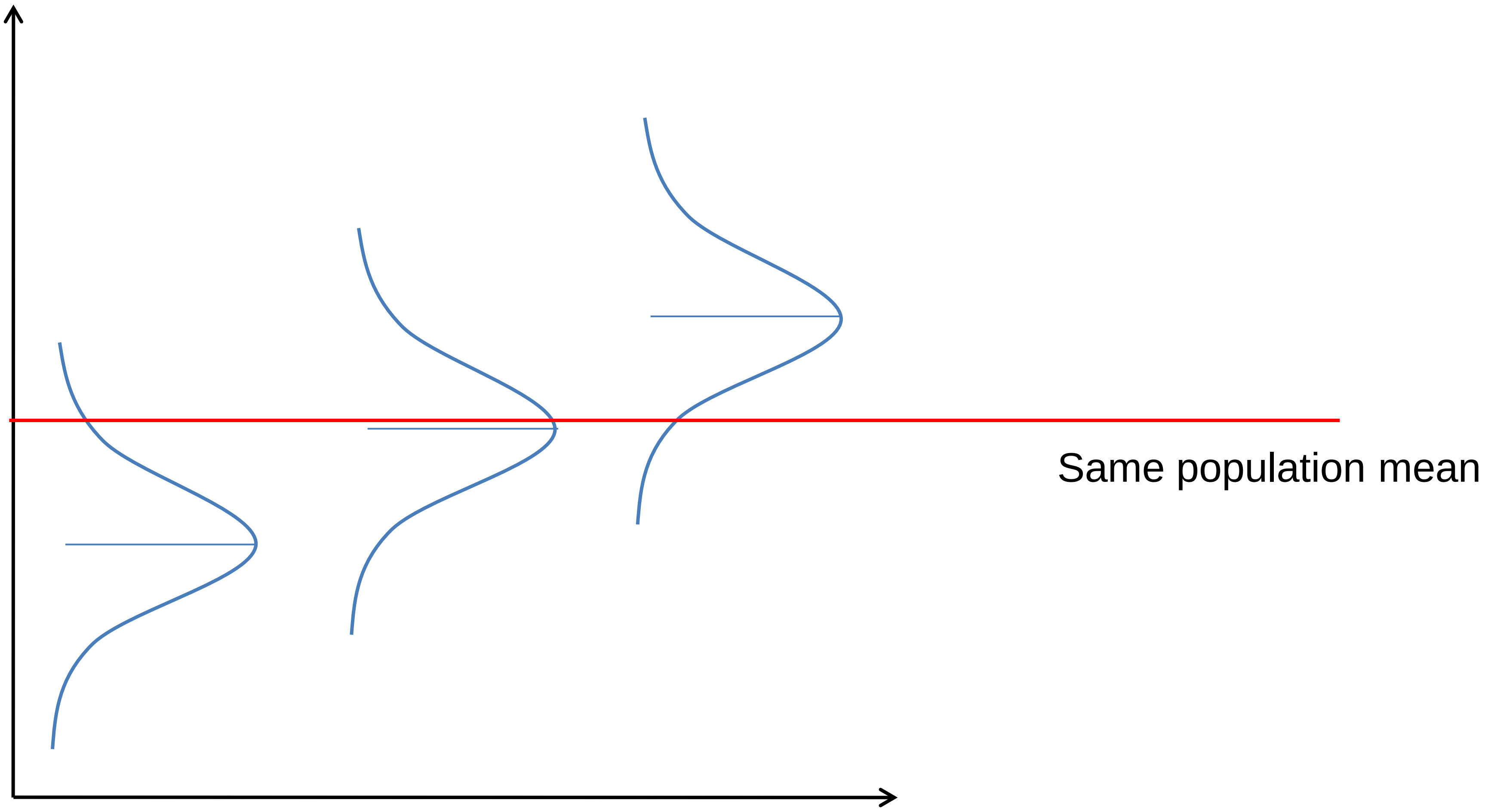
	WT_07	WT_09	WT_11
WT_07	105.782		
WT_09	0.000	105.782	
WT_11	0.000	0.000	105.782

Residuals for Covariances/Correlations/Residual Correlations

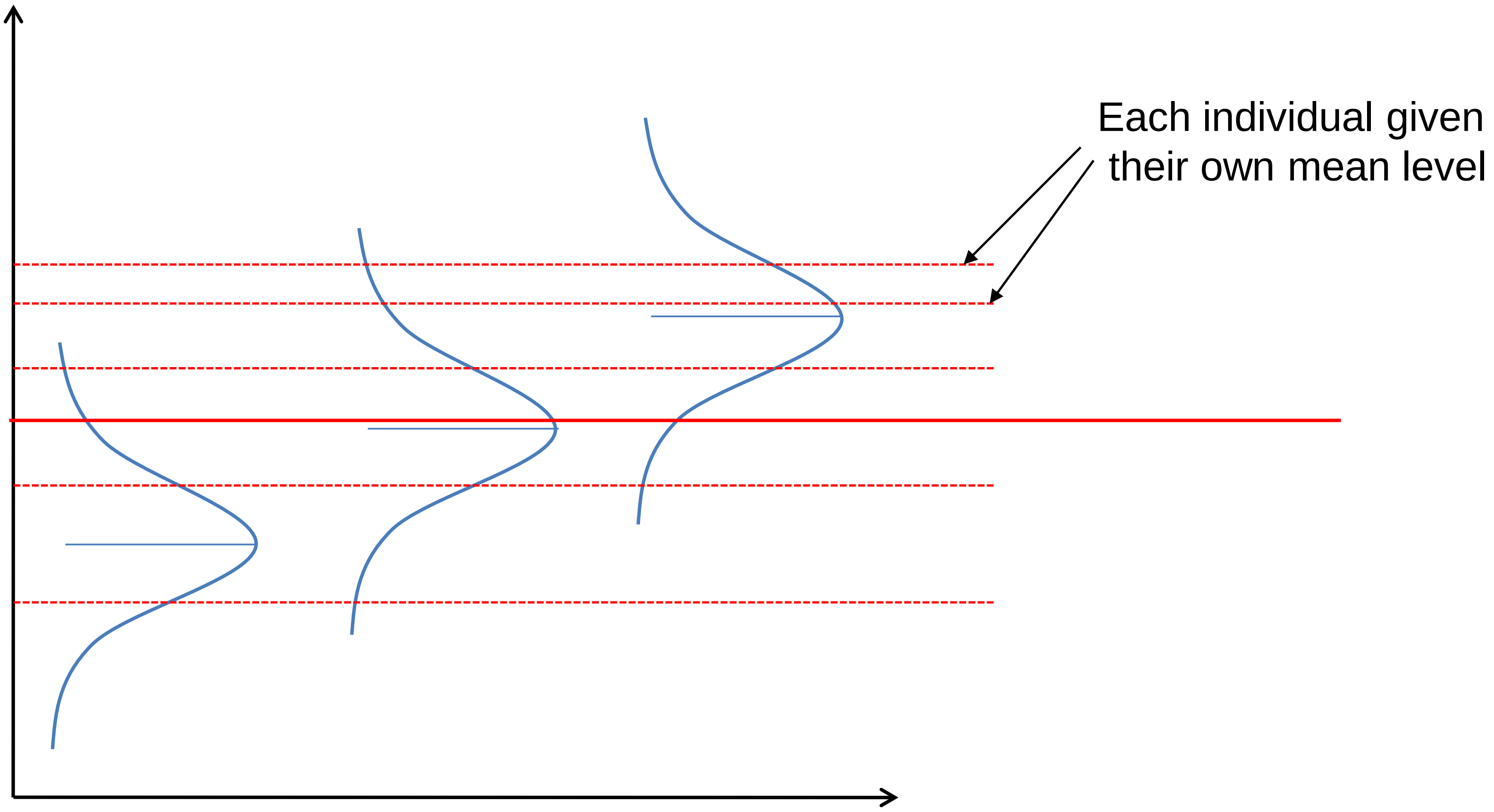
	WT_07	WT_09	WT_11
WT_07	-87.418		
WT_09	27.543	-55.995	
WT_11	35.565	63.250	-12.938

[2] Random intercept / no slope

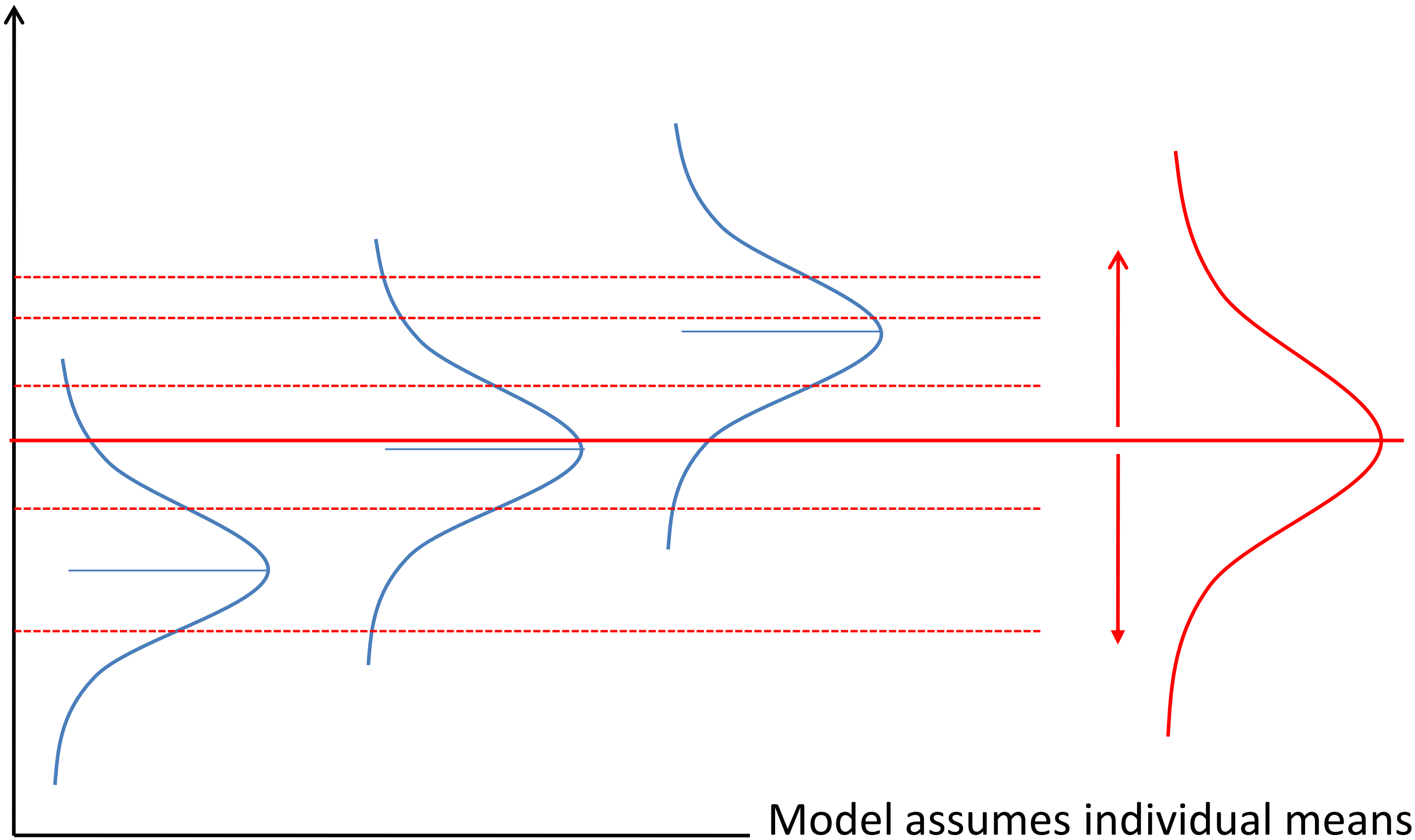
Random intercept / no slope



Random intercept / no slope

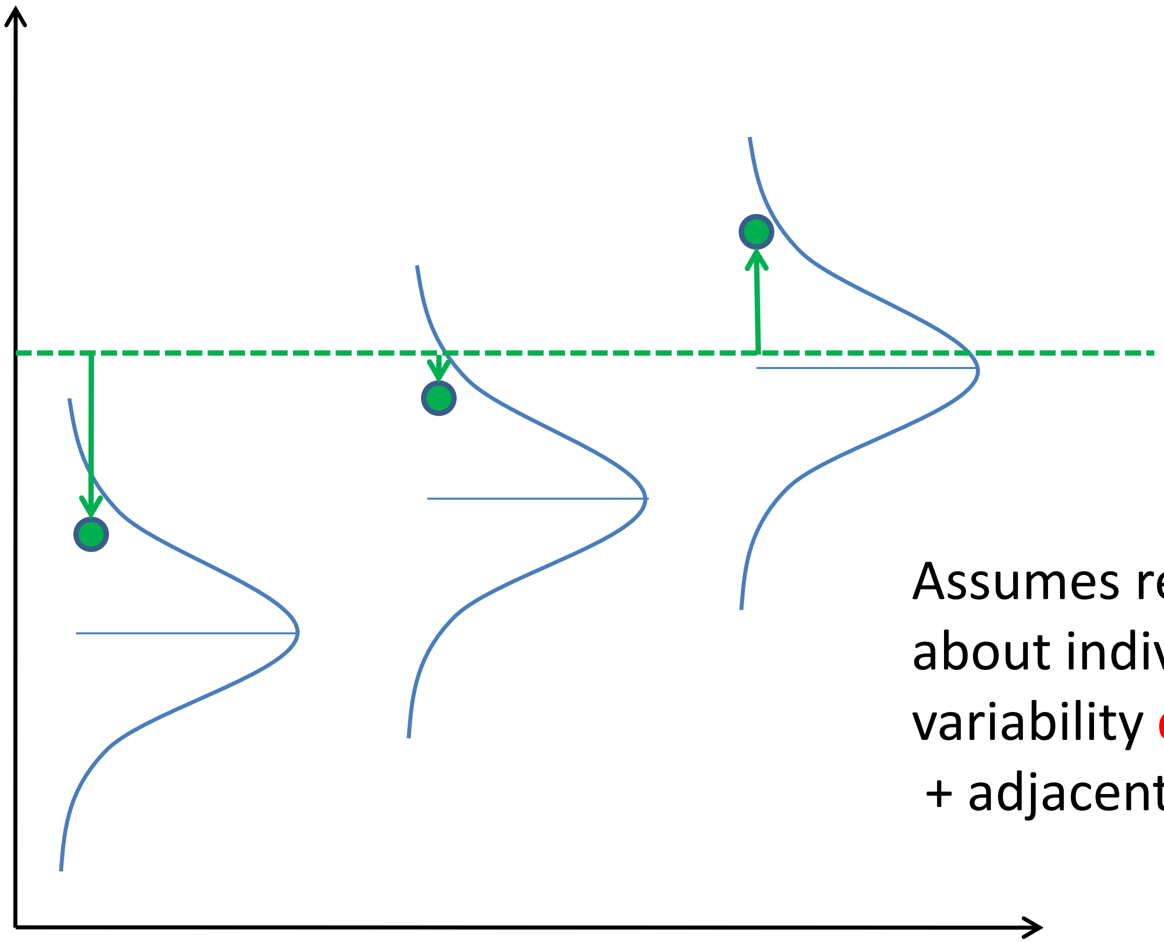


Random intercept / no slope



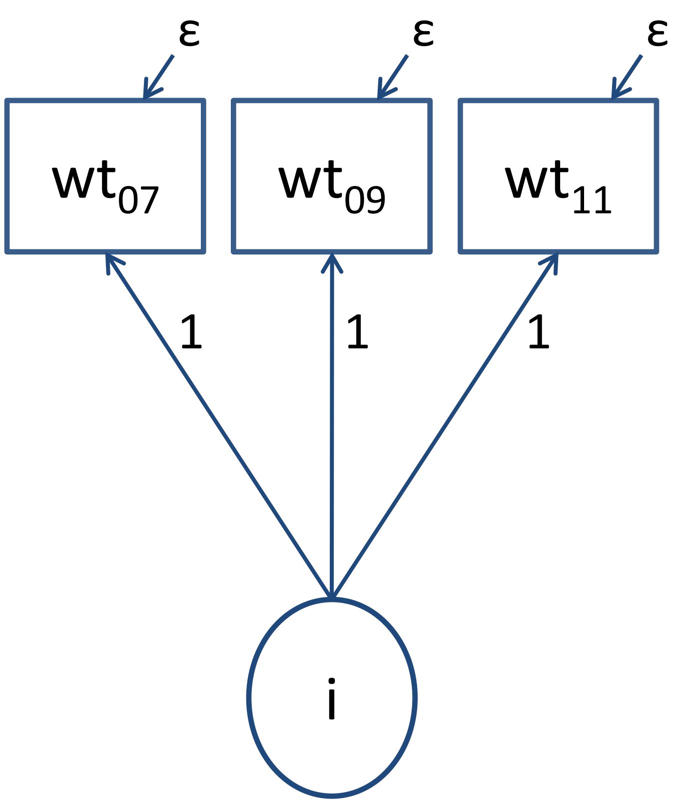
Model assumes individual means normally distributed about global mean

Random intercept / no slope



Assumes residuals are $N(0,\sigma^2)$ about individual intercepts, variability constant over time + adjacent residuals uncorrelated

Random intercept / no slope



Single growth factor
Equal residual variances

```
model:
     i | wt_07@0 wt_09@2 wt_11@4;
     wt_07 (1);
     wt_09 (1);
     wt_11 (1);
```

Which is the same as:

```
     i by wt_07@1 wt_09@1 wt_11@1;
     [wt_07@0 wt_09@0 wt_11@0 i];
     wt_07 (1);
     wt_09 (1);
     wt_11 (1);
     i;
```

Random intercept / no slope - results

```
MODEL RESULTS                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 I        |
    WT_07              1.000      0.000    999.000    999.000
    WT_09              1.000      0.000    999.000    999.000
    WT_11              1.000      0.000    999.000    999.000

 Means
    I                 34.322      0.109    315.444      0.000

 Intercepts
    WT_07              0.000      0.000    999.000    999.000
    WT_09              0.000      0.000    999.000    999.000
    WT_11              0.000      0.000    999.000    999.000

 Variances
    I                 16.061      1.148     13.986      0.000

 Residual Variances
    WT_07             89.722      1.440     62.314      0.000
    WT_09             89.722      1.440     62.314      0.000
    WT_11             89.722      1.440     62.314      0.000
```

Same mean as before

Random intercept / no slope - results

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
I |				
WT_07	1.000	0.000	999.000	999.000
WT_09	1.000	0.000	999.000	999.000
WT_11	1.000	0.000	999.000	999.000
Means				
I	34.322	0.109	315.444	0.000
Intercepts				
WT_07	0.000	0.000	999.000	999.000
WT_09	0.000	0.000	999.000	999.000
WT_11	0.000	[illegible]	[illegible]	999.000
Variances				
I	16.061	1.148	13.986	0.000
Residual Variances				
WT_07	89.722	[illegible]	[illegible]	0.000
WT_09	89.722	[illegible]	[illegible]	0.000
WT_11	89.722	[illegible]	[illegible]	0.000

Between subject variance

Within subject variance

Random intercept / no slope - results

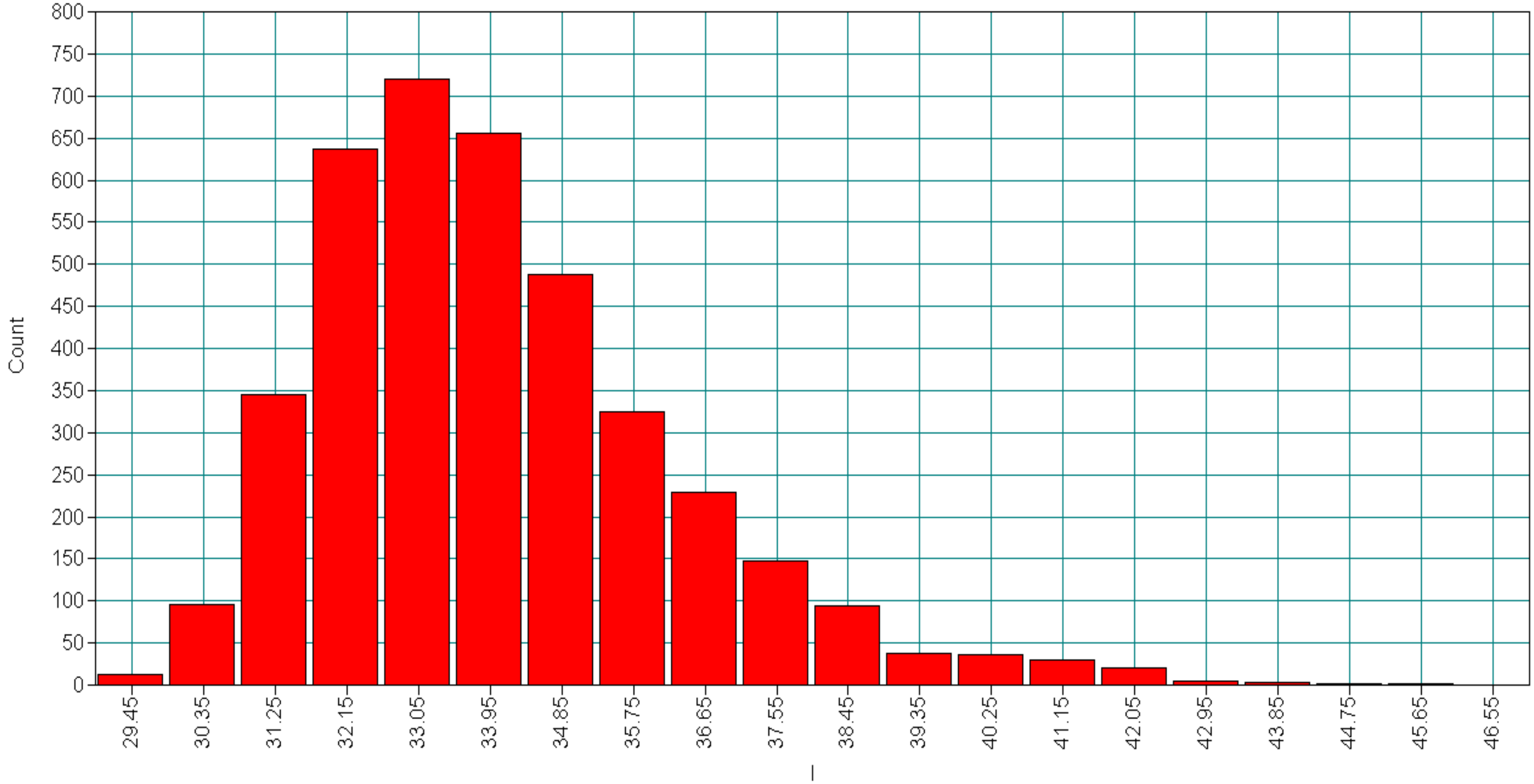
MODEL RESULTS	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
I |				
WT_07	1.000			
WT_09	1.000			
WT_11	1.000			
Means				
I	34.322			
Intercepts				
WT_07	0.000	0.000	999.000	999.000
WT_09	0.000	0.000	999.000	999.000
WT_11	0.000	[illegible]	[illegible]	999.000
Variances				
I	16.061	1.148	13.986	0.000
Residual Variances				
WT_07	89.722	[illegible]	[illegible]	0.000
WT_09	89.722	[illegible]	[illegible]	0.000
WT_11	89.722	[illegible]	[illegible]	0.000

ICC
= 16.061/(16.061 + 89.722)
= 0.152

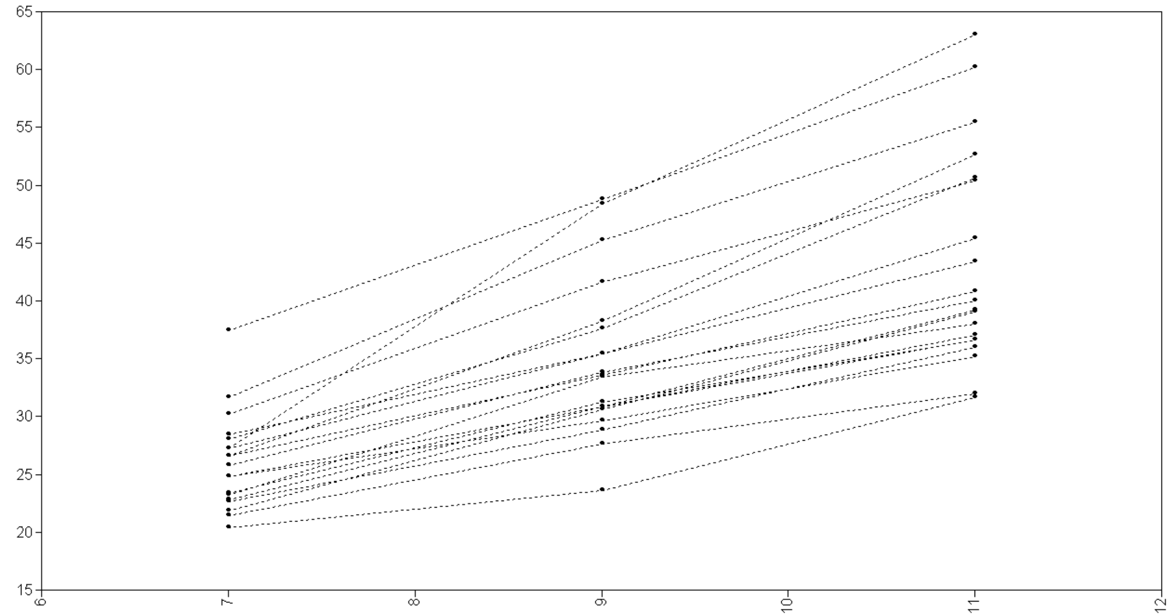
Between subject variance

Within subject variance

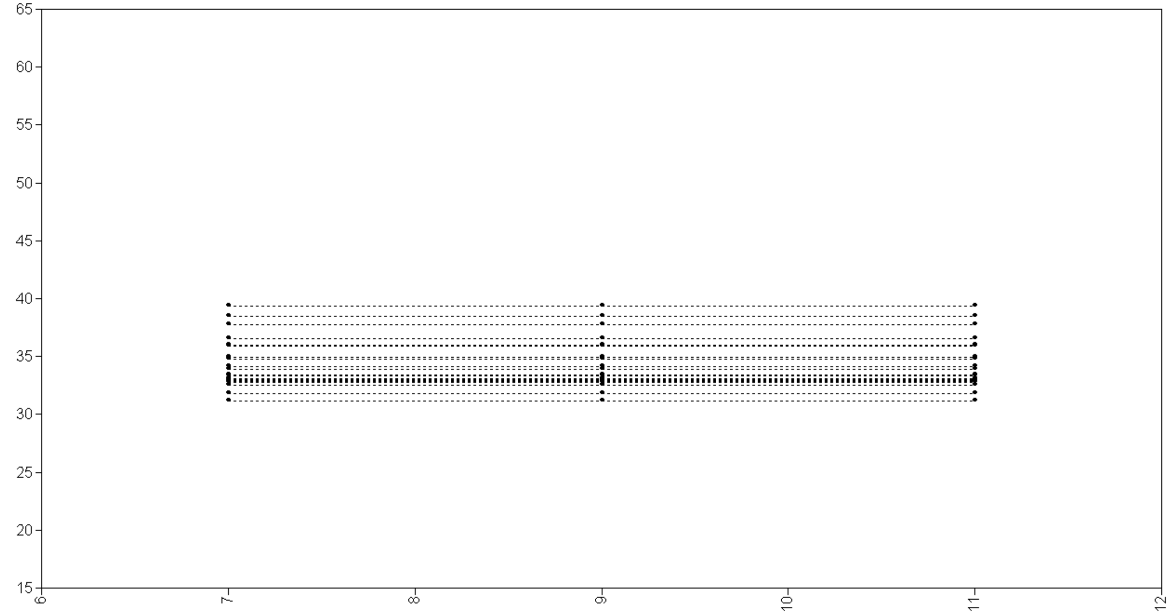
Histogram of intercept factor



Observed data (cases 1-20)



Estimated data (cases 1-20)



Random intercept / no slope - residuals

Means

WT_07	WT_09	WT_11
25.532	34.219	43.214

Model Estimated Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
34.322	34.322	34.322

Residuals for Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
-8.790	-0.103	8.893

No change here!
We’ve played with the variance but not affected the mean structure

Random intercept / no slope - residuals

Covariances

	WT_07	WT_09
WT_07	18.365	
WT_09	27.543	49.787
WT_11	35.565	63.250

Model Estimated Covariances/Correlations

	WT_07	WT_09
WT_07	105.783	
WT_09	16.061	105.783
WT_11	16.061	16.061

This model has assigned an *exchangeable* covariance structure. We have allowed each individual's set of measurements to be correlated.

We would expect in a growth situation that adjacent measures would be more highly correlated

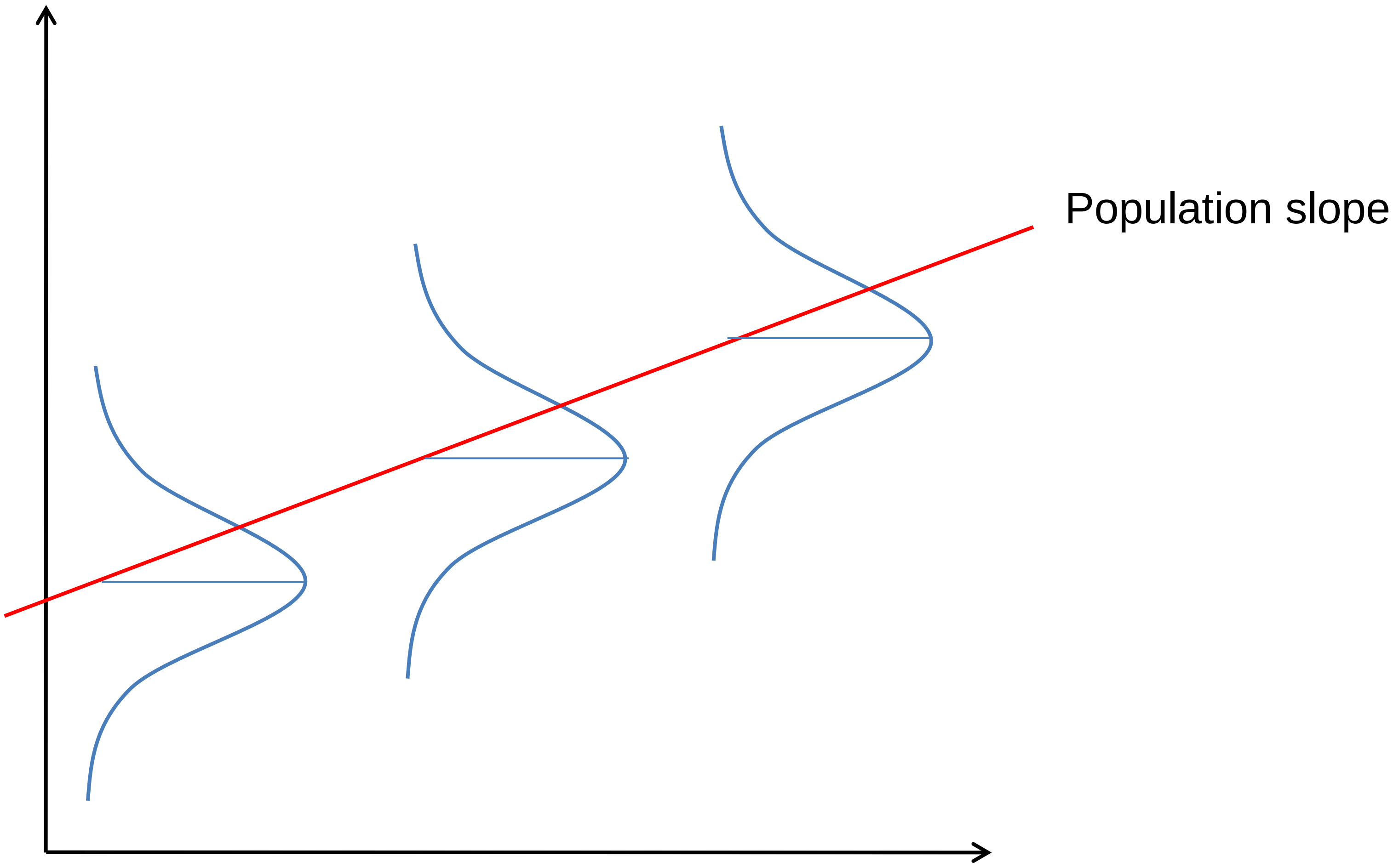
It is also common for variances to increase as the kids' weights fan outwards – unlike here

Residuals for Covariances/Correlations/Residual Correlations

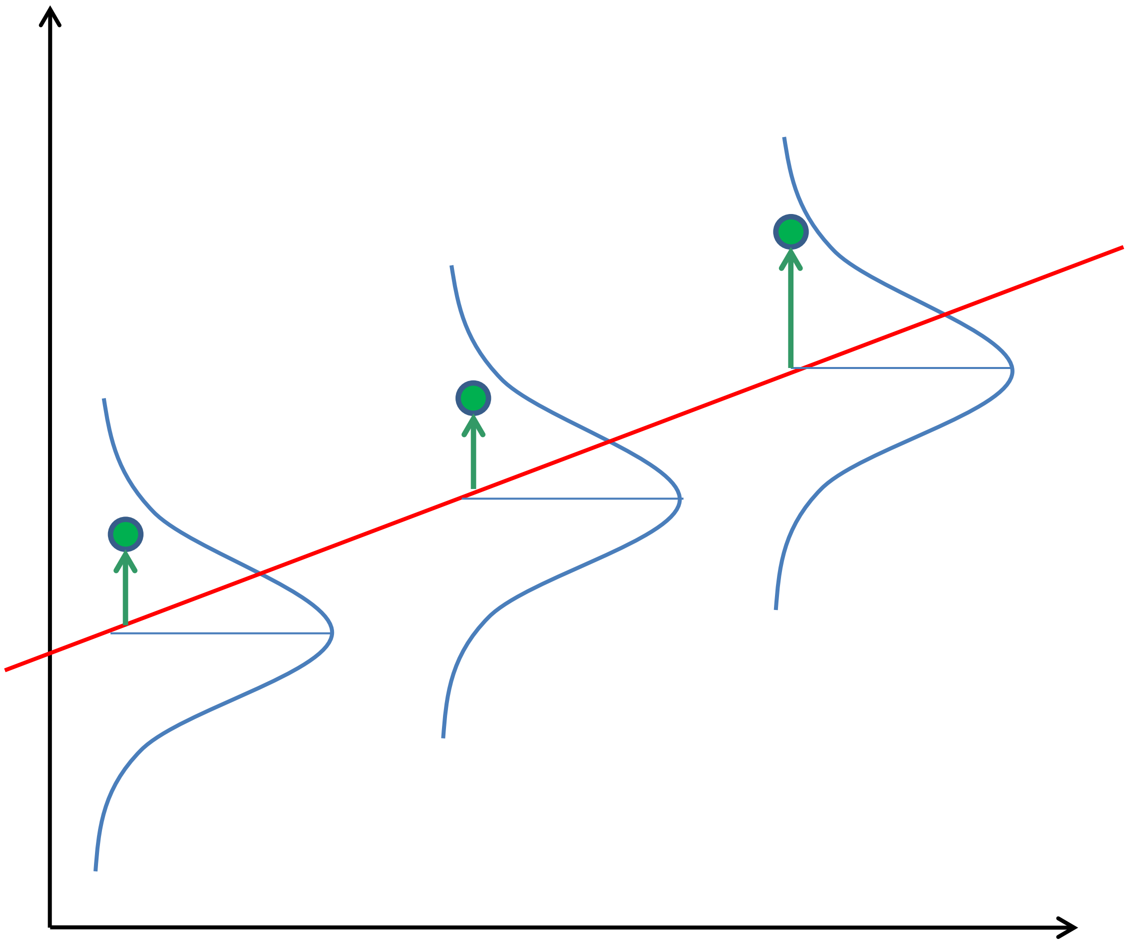
	WT_07	WT_09	WT_11
WT_07	-87.418		
WT_09	11.483	-55.996	
WT_11	19.504	47.189	-12.938

[3] Fixed intercept / fixed slope

Fixed intercept / fixed slope



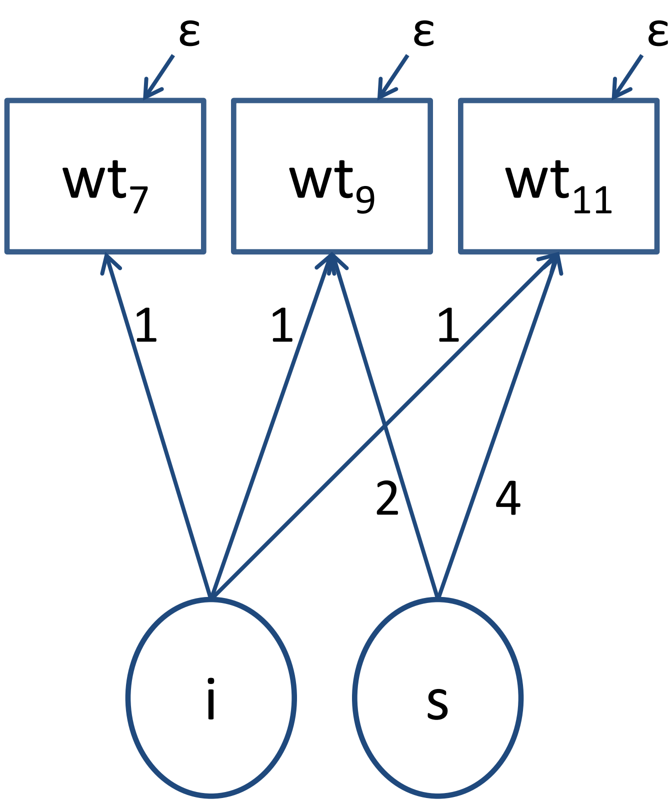
Fixed intercept / fixed slope



Model assumes
Uncorrelated residuals

Clearly violated

Fixed intercept / fixed slope

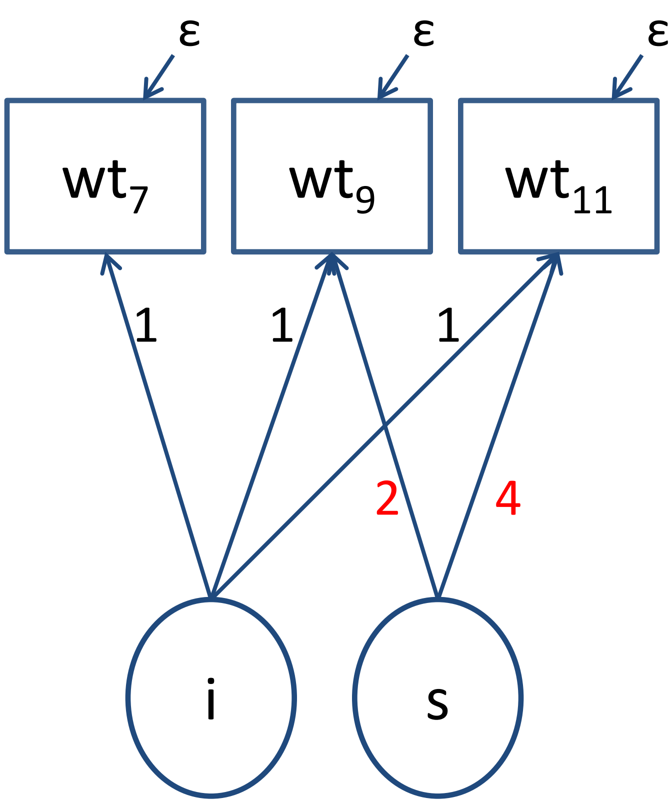



```
model:
  i s | wt_07@0 wt_09@2 wt_11@4;
  wt_07 (1);
  wt_09 (1);
  wt_11 (1);
  i@0 s@0;
  s with s@0;
```

Which is the same as:

```
i by wt_07@1 wt_09@1 wt_11@1;
s by wt_07@0 wt_09@2 wt_11@4;
wt_07 (1) wt_09 (1) wt_11 (1);
[wt_07@0 wt_09@0 wt_11@0 i s];
i@0 s@0;
s with s@0;
```

Choice of loadings for SLOPE



It is traditional for the intercept factor to have a unit loading on each repeated measure

Here we have used loadings of 0/2/4 for the slope factor since the repeated measures are 2 years apart.

E.g. expected weight at age 11
= intercept plus 4*gradient

Alternative loadings for slope would be 7/11/13, 0/24/48, … as long as the relative spacing preserved.

The interpretation of i has now changed

Fixed intercept / fixed slope - results

```
MODEL RESULTS                                              Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 <snip>

 I        WITH
    S                  0.000      0.000    999.000    999.000

 Means
    I                 25.480      0.107    237.416      0.000
    S                  4.421      0.042    106.352      0.000

 Intercepts
    WT_07              0.000      0.000    999.000    999.000
    WT_09              0.000                            0.000
    WT_11              0.000                            0.000

 Variances
    I                  0.000      0.000    999.000    999.000
    S                  0.000      0.000    999.000    999.000

 Residual Variances
    WT_07             53.671      0.703     76.318      0.000
    WT_09             53.671      0.703     76.318      0.000
    WT_11             53.671      0.703     76.318      0.000
```

(25.532 + 34.219 + 43.214)/3 - 2*4.421

(43.214 - 25.532)/4

`regress wt time` (data in long-format)

```
      Source |       SS       df       MS              Number of obs =   11649
-------------+------------------------------           F(  1, 11647) =11308.80
       Model |  607054.836     1  607054.836           Prob > F      =  0.0000
    Residual |  625209.224 11647  53.6798509           R-squared     =  0.4926
-------------+------------------------------           Adj R-squared =  0.4926
       Total |  1232264.06 11648  105.791901           Root MSE      =  7.3267

------------------------------------------------------------------------------
          wt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   4.420641   .0415697   106.34   0.000     4.339158    4.502125
       _cons |  -5.464242    .380236   -14.37   0.000    -6.209568   -4.718915
------------------------------------------------------------------------------
```

Of course, we'd never do this as we've totally ignored the **clustering** within individual

Fixed intercept / fixed slope - residuals

Means

WT_07	WT_09	WT_11
25.532	34.219	43.214

Model Estimated Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
25.480	34.322	43.163

Residuals for Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
0.051	-0.103	0.051

Residuals for means have much improved

Seems that population growth well approximated by linear slope

Fixed intercept / fixed slope - residuals

Covariances

	WT_07	WT_09	W
WT_07	18.365		
WT_09	27.543	49.787	
WT_11	35.565	63.250	9

We are back to assuming no covariances so no surprise that residuals are high

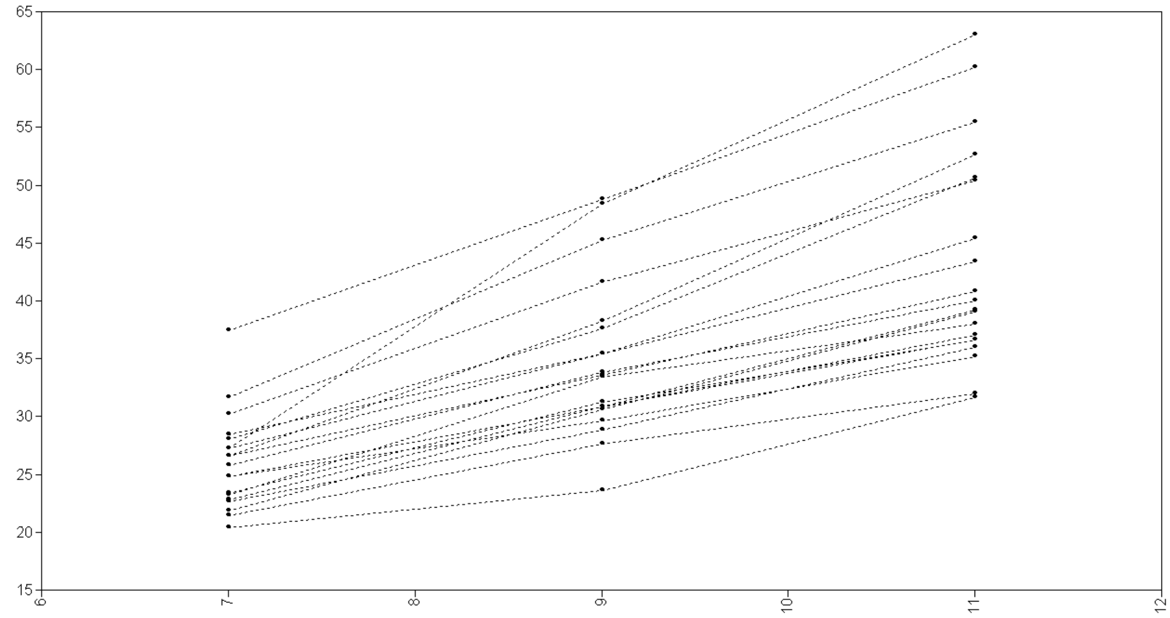
Model Estimated Covariances/Correlations/Residual Correlations

	WT_07	WT_09	WT_11
WT_07	53.671		
WT_09	**0.000**	53.671	
WT_11	**0.000**	**0.000**	53.671

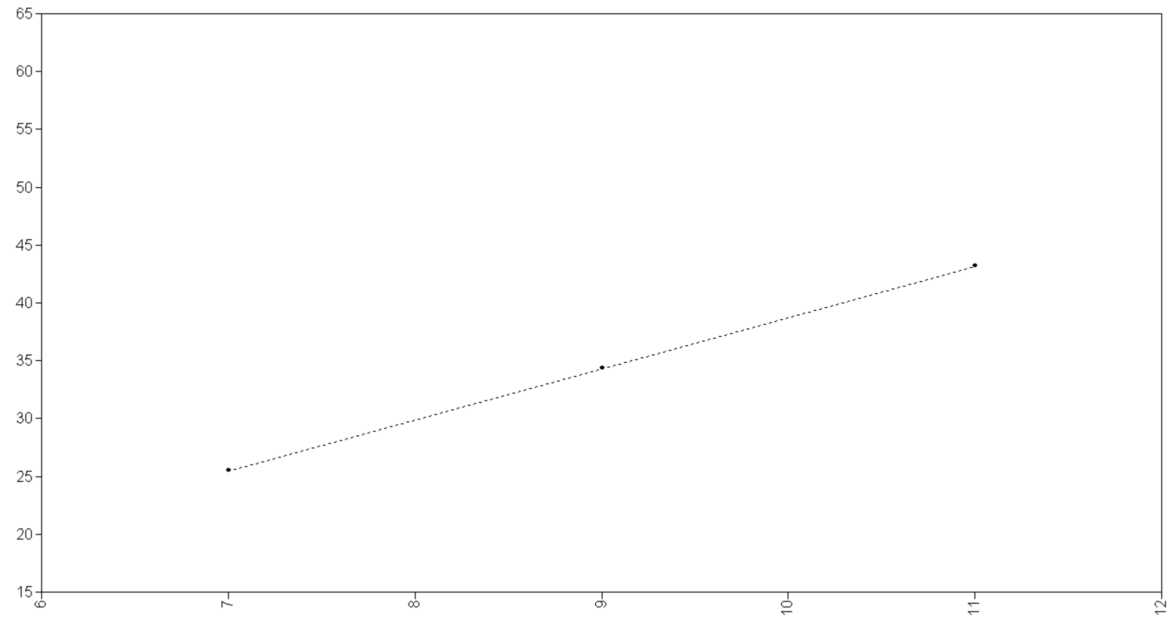
Residuals for Covariances/Correlations/Residual Correlations

	WT_07	WT_09	WT_11
WT_07	-35.306		
WT_09	27.543	-3.884	
WT_11	35.565	63.250	39.174

Observed data
(cases 1-20)

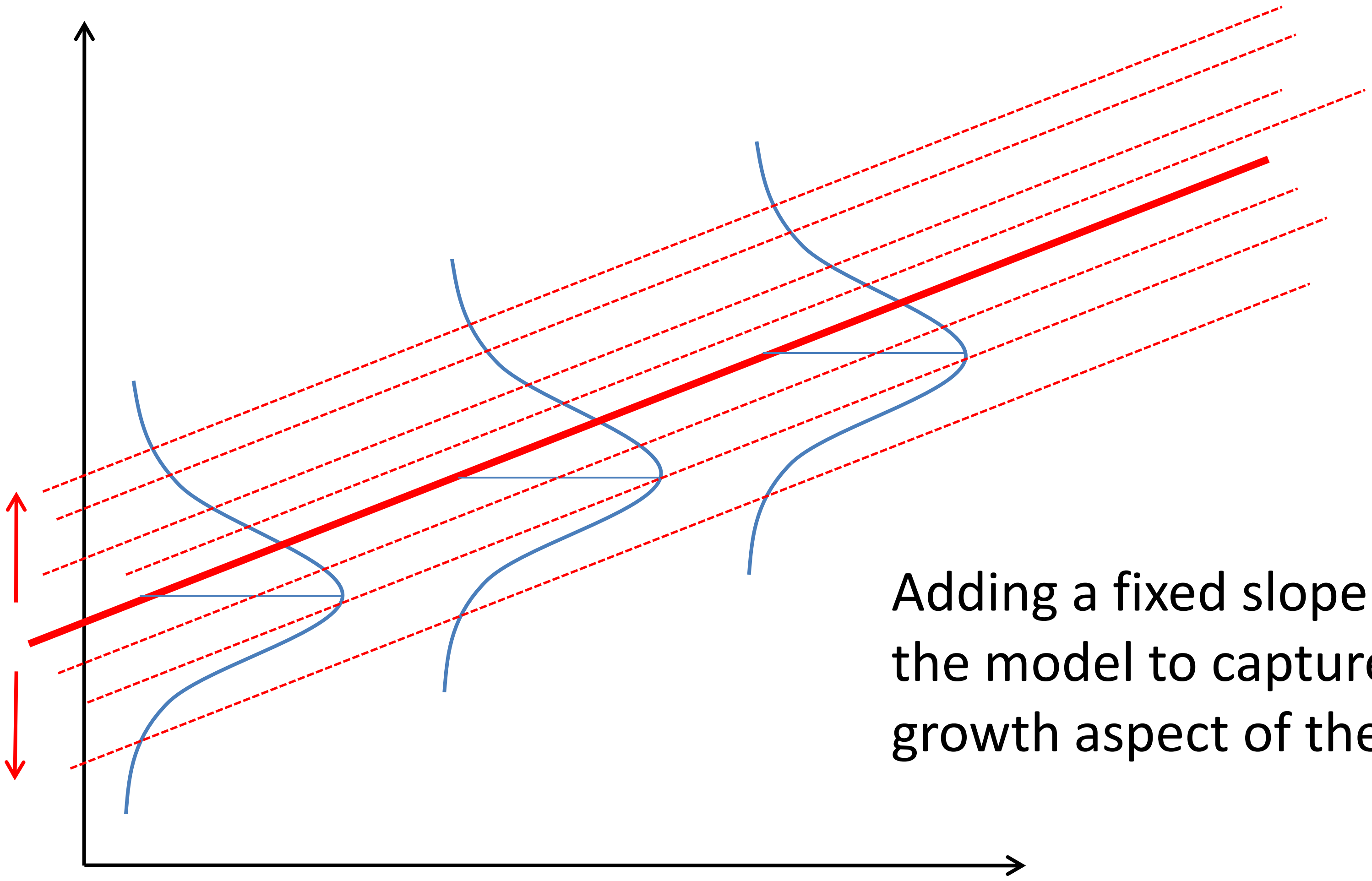


Estimated data
(cases 1-20)



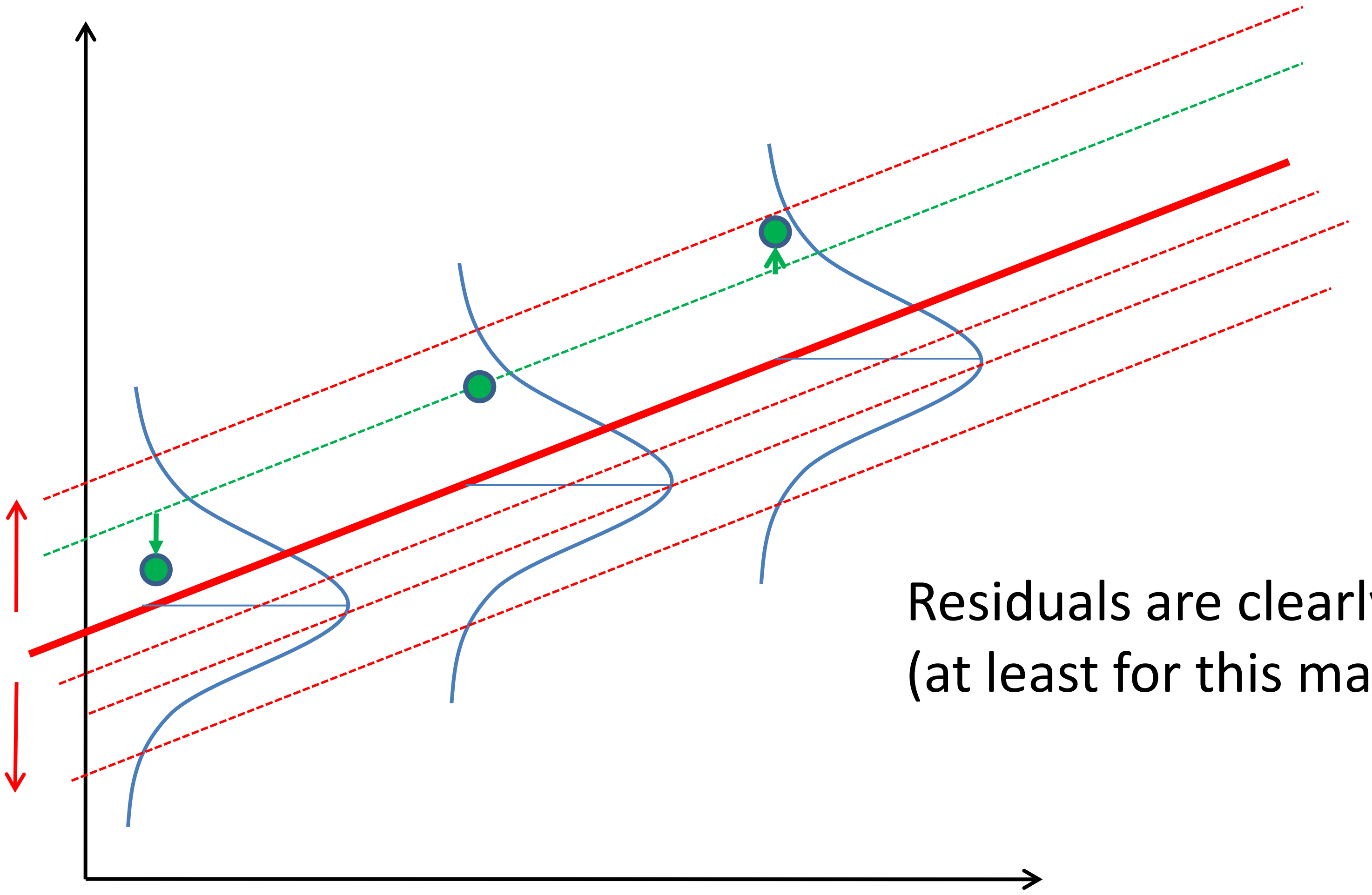
[4] Random intercept / fixed slope

Random intercept / fixed slope



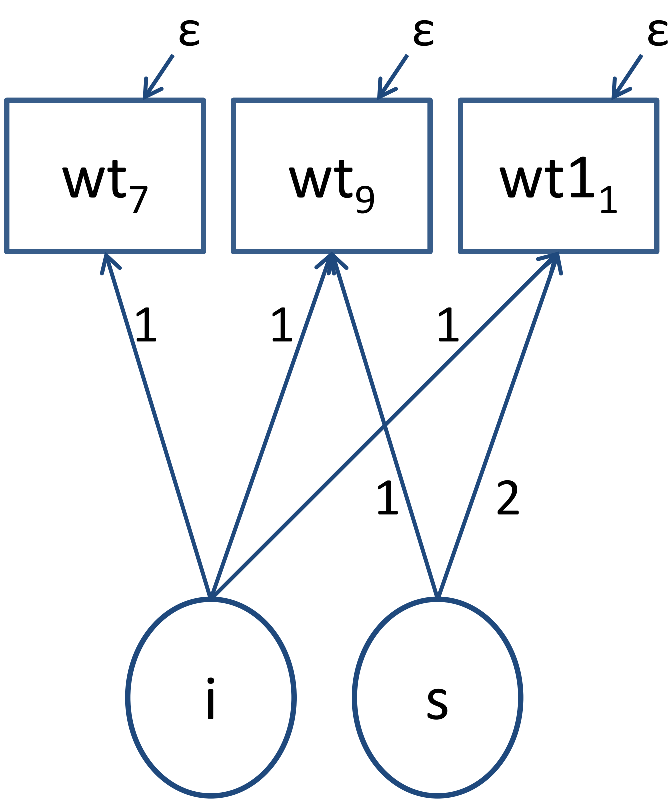
Adding a fixed slope has allowed the model to capture the growth aspect of the data

Random intercept / fixed slope



Residuals are clearly SMALLER
(at least for this made-up person)

Random intercept / fixed slope




```
model:
   i s | wt_07@0 wt_09@2 wt_11@4;
   wt_07 (1);
   wt_09 (1);
   wt_11 (1);
   i;
   s@0;           ! Slope has no variance
   i with s@0;    ! Hence no covariance
```

Random intercept / fixed slope - results

```
MODEL RESULTS
                                                      Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

<snip>

 I        WITH
    S                  0.000      0.000     999.(

 Means
    I                 25.480      0.115    220.726      0.000
    S                  4.421      0.019    229.218      0.000

 Intercepts
    WT_07              0.000      0.000    999.000    999.000
    WT_09              0.000      0.000    999.00
    WT_11              0.000      0.000    999.00

 Variances
    I                 42.117      1.045     40.30
    S                  0.000      0.000    999.00

 Residual Variances
    WT_07             11.554      0.185     62.313      0.000
    WT_09             11.554      0.185     62.313      0.000
    WT_11             11.554      0.185     62.313      0.000
```

Mean structure unaffected by allowing intercepts to vary

Previous residual variance has now been partitioned

The majority is now due to variation in starting weight

Random intercept / fixed slope - residuals

Means

WT_07	WT_09	WT_11
25.532	34.219	43.214

Model Estimated Means/Intercepts/Thresholds

No change here!

WT_07	WT_09	WT_11
25.480	34.322	43.163

Residuals for Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
0.051	-0.103	0.051

Random intercept / fixed slope - residuals

We are back to an
exchangeable covariance structure.

Compared with the
random intercept / no slope model
the estimated covariances are
higher and hence corresponding
residuals lower

Covariances

	WT_07	WT_09
WT_07	18.365	
WT_09	27.543	49.787
WT_11	35.565	63.250

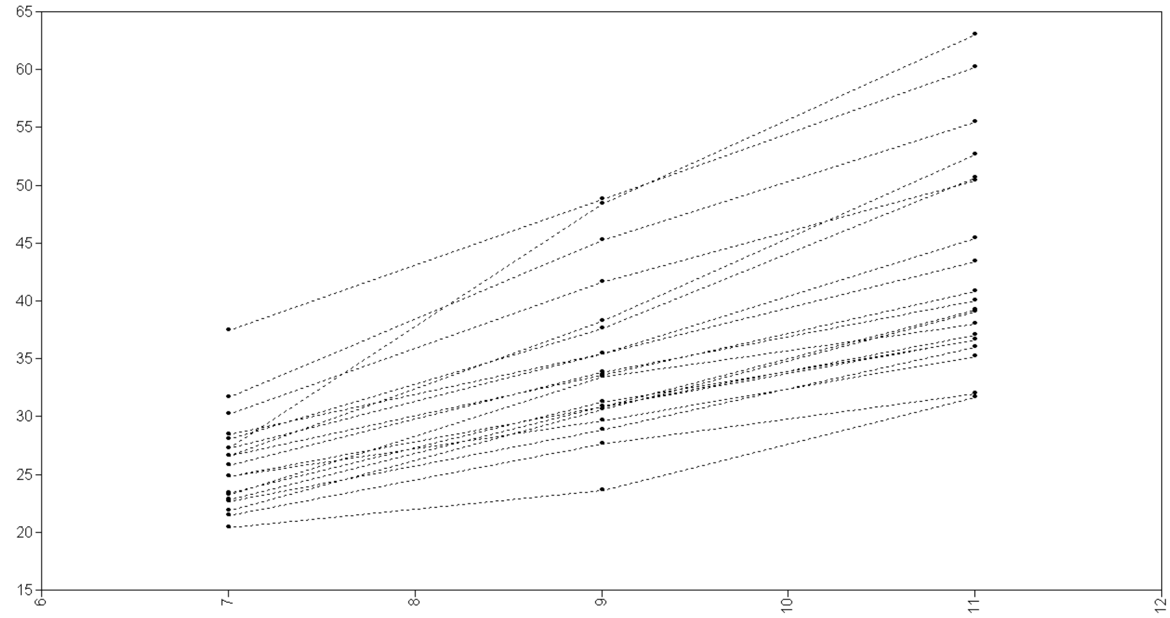
Model Estimated Covariances/Correlati

	WT_07	WT_09	
WT_07	53.671		
WT_09	**42.117**	53.671	
WT_11	**42.117**	**42.117**	53.671

Residuals for Covariances/Correlations/Residual Correlations

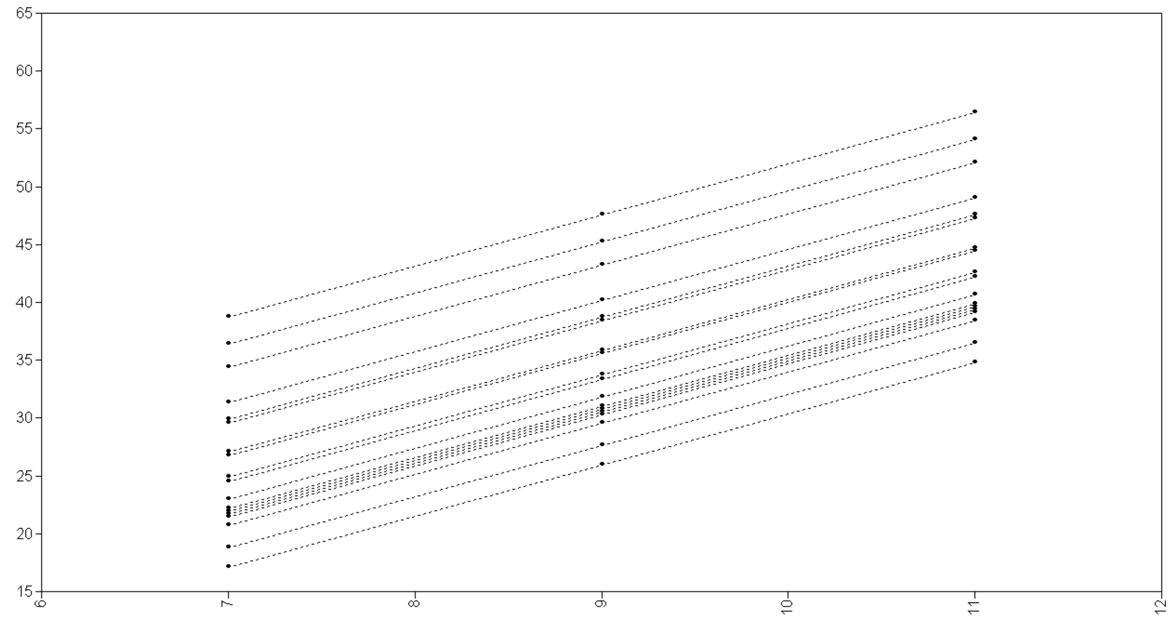
	WT_07	WT_09	WT_11
WT_07	-35.306		
WT_09	-14.574	-3.884	
WT_11	-6.552	21.133	39.174

Observed data (cases 1-20)



Things are starting to look much better!

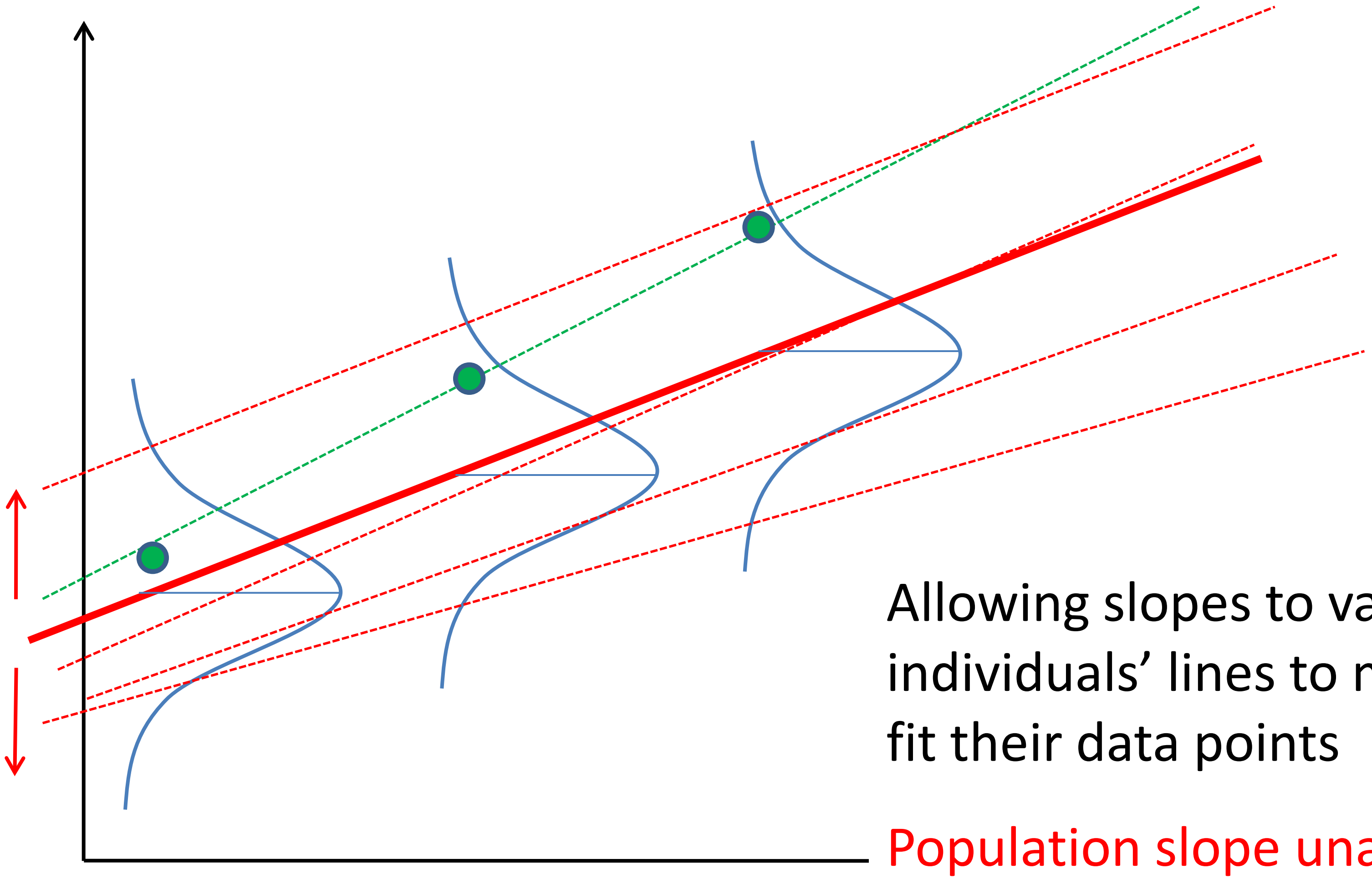
Estimated data (cases 1-20)



[5] Random intercept / random slope

(Standard linear growth model)

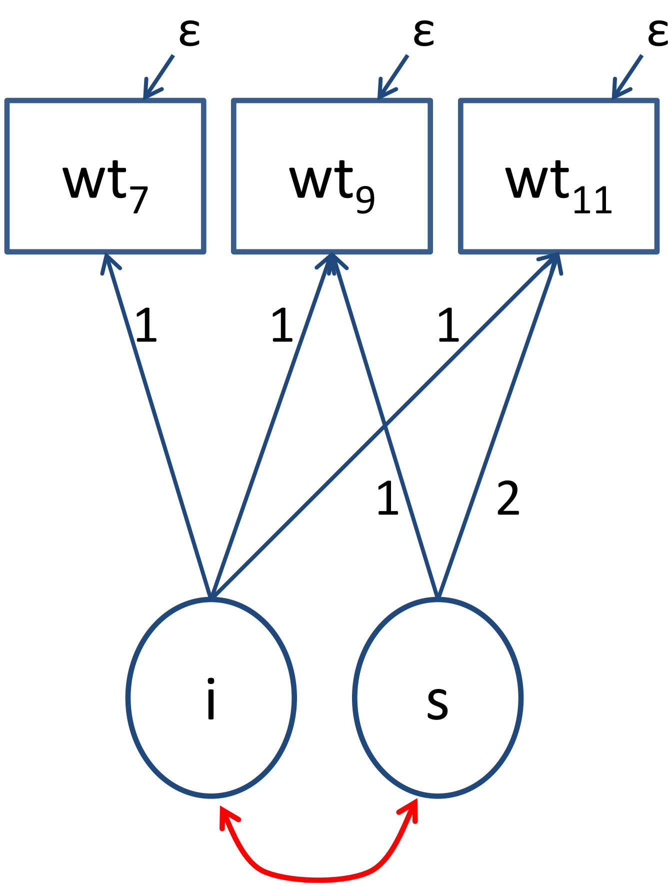
Random intercept / random slope



Allowing slopes to vary allows individuals' lines to more closely fit their data points

Population slope unaffected

Random intercept / random slope




```
model:
   i s | wt_07@0 wt_09@2 wt_11@4;
   wt_07 (1);
   wt_09 (1);
   wt_11 (1);
   i s;
   i with s;
```

Random intercept / random slope - results

```
MODEL RESULTS                                            Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

<snip>

 S        WITH
    I                  4.939      0.131     37.652      0.000

 Means
    I                 25.480      0.070    361.802      0.000
    S                  4.421      0.025    174.049      0.000

 Intercepts
    WT_07              0.000      0.000    999.000    999.000
    WT_09              0.000      0.000    999.000    999.000
    WT_11              0.000      0.000    999.000    999.000

 Variances
    I                 16.702      0.441     37.880      0.000
    S                  2.121      0.058     36.886      0.000

 Residual Variances
    WT_07              3.068      0.070     44.062      0.000
    WT_09              3.068      0.070     44.062      0.000
    WT_11              3.068      0.070     44.062      0.000
```

Random intercept / random slope - residuals

Means

WT_07	WT_09	WT_11
25.532	34.219	43.214

Model Estimated Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
25.480	34.322	43.163

Residuals for Means/Intercepts/Thresholds

WT_07	WT_09	WT_11
0.051	-0.103	0.051

Random intercept / random slope - residuals

Covariances

	WT_07	WT_09
WT_07	18.365	
WT_09	27.543	49.787
WT_11	35.565	63.250

Covariance matrix no longer exchangeable

Variances allowed to increase with time

Vast improvement in residuals

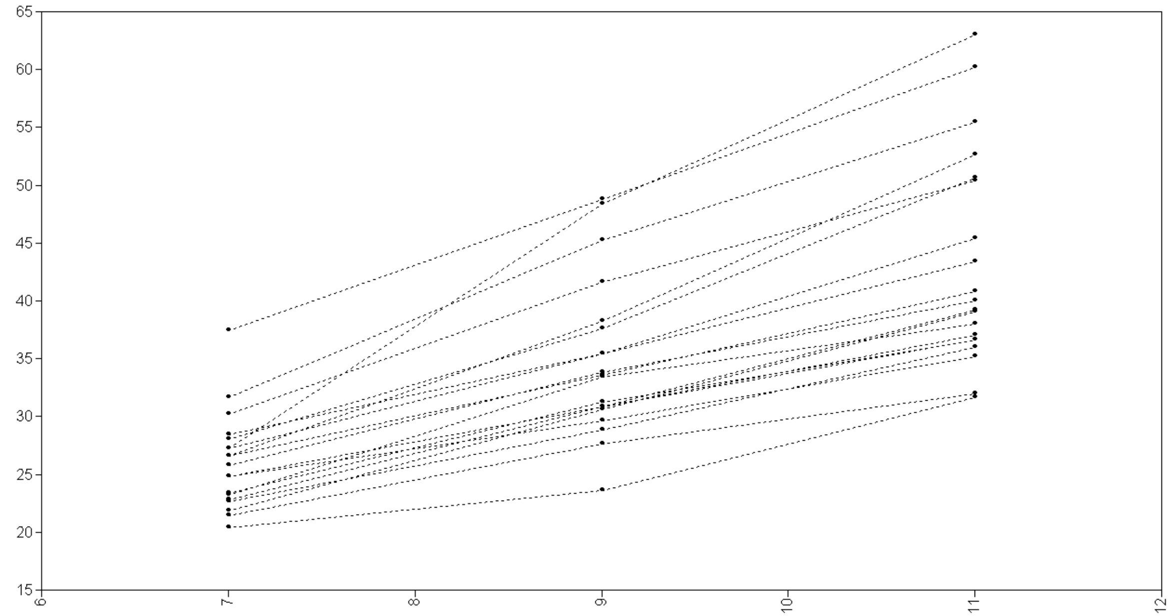
Model Estimated Covariances/Correlations

	WT_07	WT_09	WT_11
WT_07	19.770		
WT_09	**26.581**	48.014	
WT_11	**36.460**	**63.310**	93.228

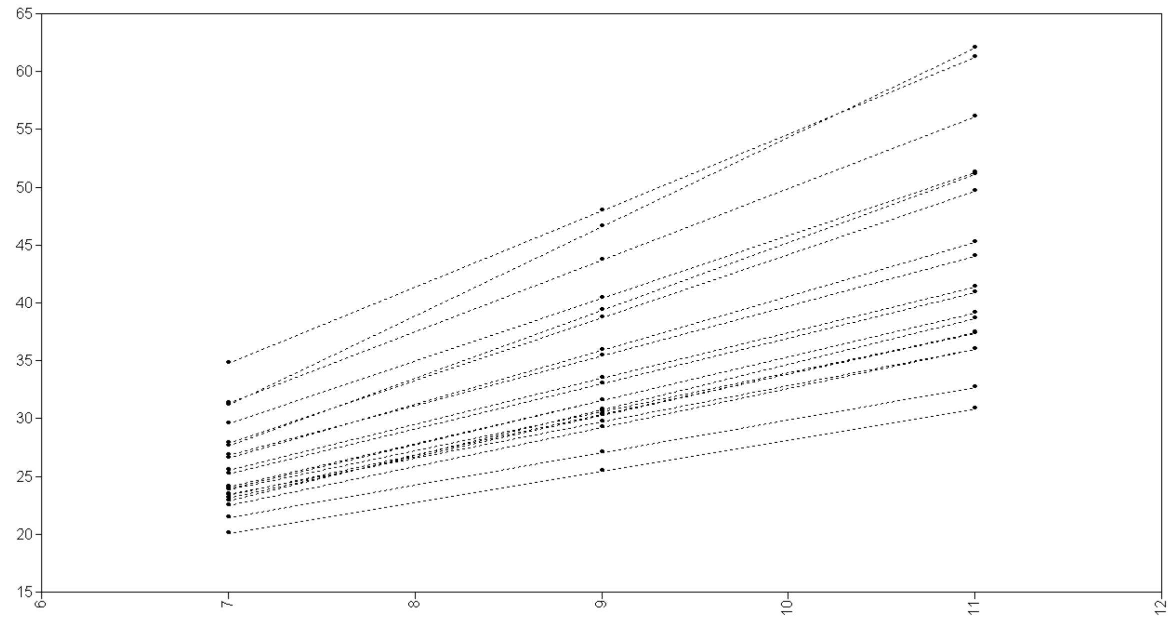
Residuals for Covariances/Correlations/Residual Correlations

	WT_07	WT_09	WT_11
WT_07	**-1.406**		
WT_09	**0.962**	**1.773**	
WT_11	**-0.895**	**-0.060**	**-0.384**

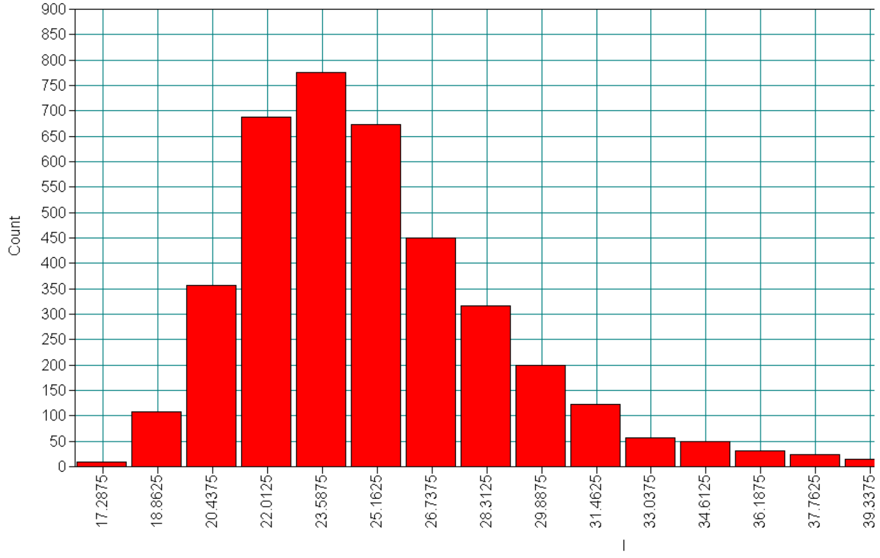
Observed data
(cases 1-20)

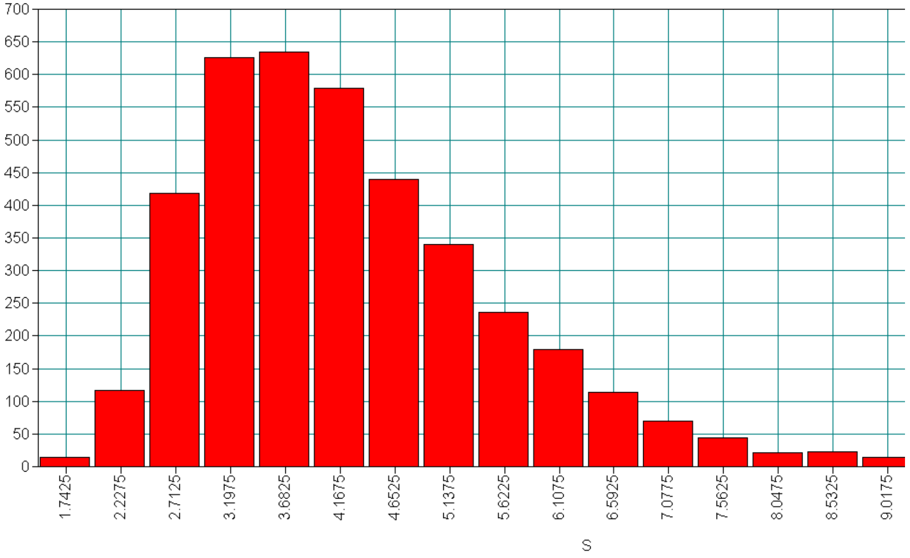


Estimated data
(cases 1-20)



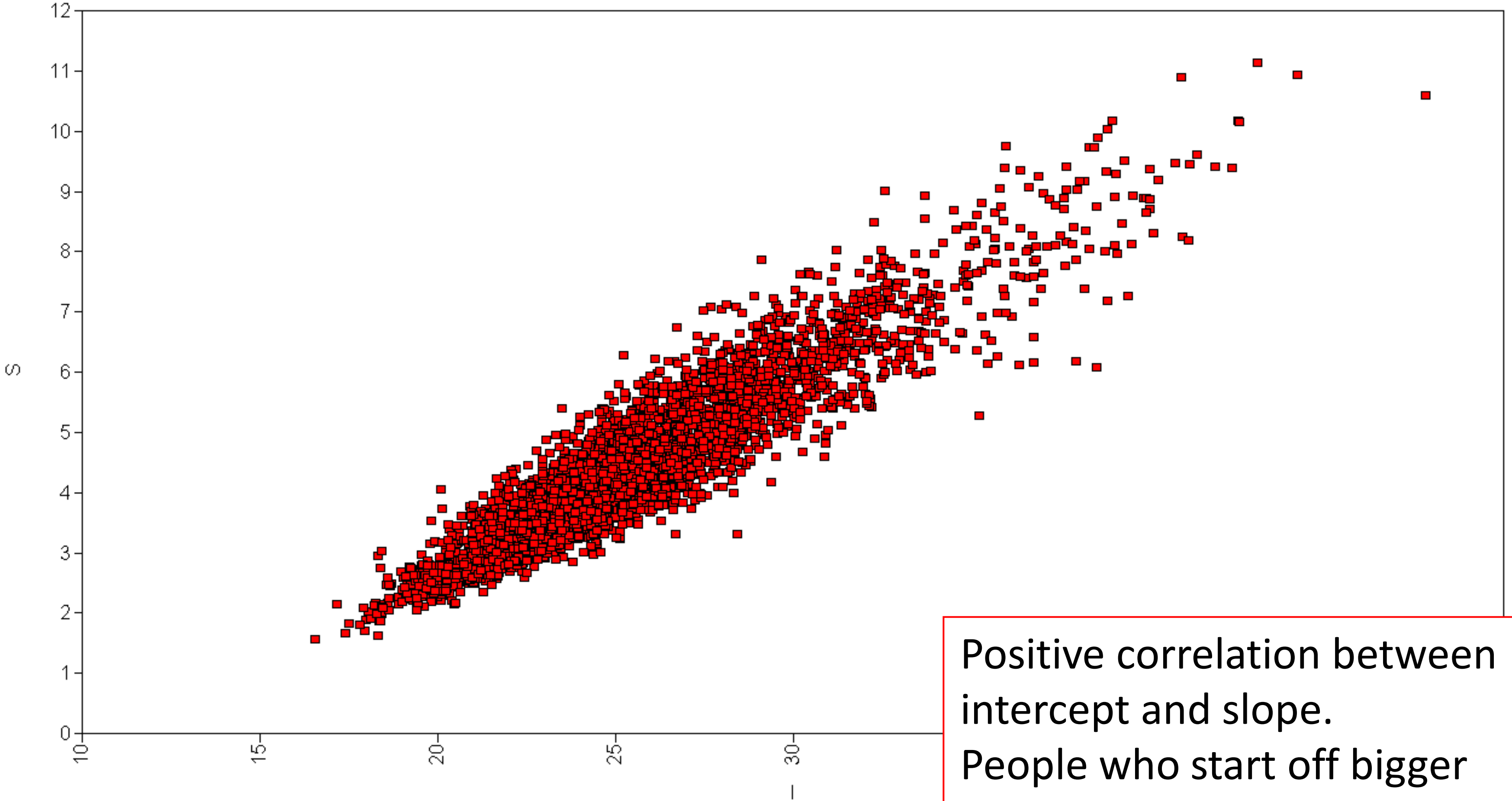
Histogram of intercept / slope factors





High variability in intercept and slope

Scatterplot of I versus S



Positive correlation between intercept and slope.
People who start off bigger grow faster

Results summary – growth factor means

	Mean(i)	Mean(s)
Fixed intercept / no slope	34.32 (0.095)	-
Random intercept / no slope	34.32 (0.109)	-
Fixed intercept / fixed slope	25.48 (0.107)	4.42 (0.042)
Random intercept / fixed slope	25.48 (0.115)	4.42 (0.019)
Random intercept / random slope	25.48 (0.070)	4.42 (0.025)

Notice improvement in precision

Results summary – (co)variances

	Var(i)	Var(s)	Cov(i,s)	Res var
Fixed intercept / no slope	-	-	-	105.8 (1.39)
Random intercept / no slope	16.06 (1.15)	-	-	89.72 (1.44)
Fixed intercept / fixed slope	-	-	-	53.67 (0.70)
Random intercept / fixed slope	42.12 (1.05)	-	-	11.56 (0.19)
Random intercept / random slope	16.70 (0.44)	2.12 (0.06)	4.94 (0.13)	3.07 (0.07)

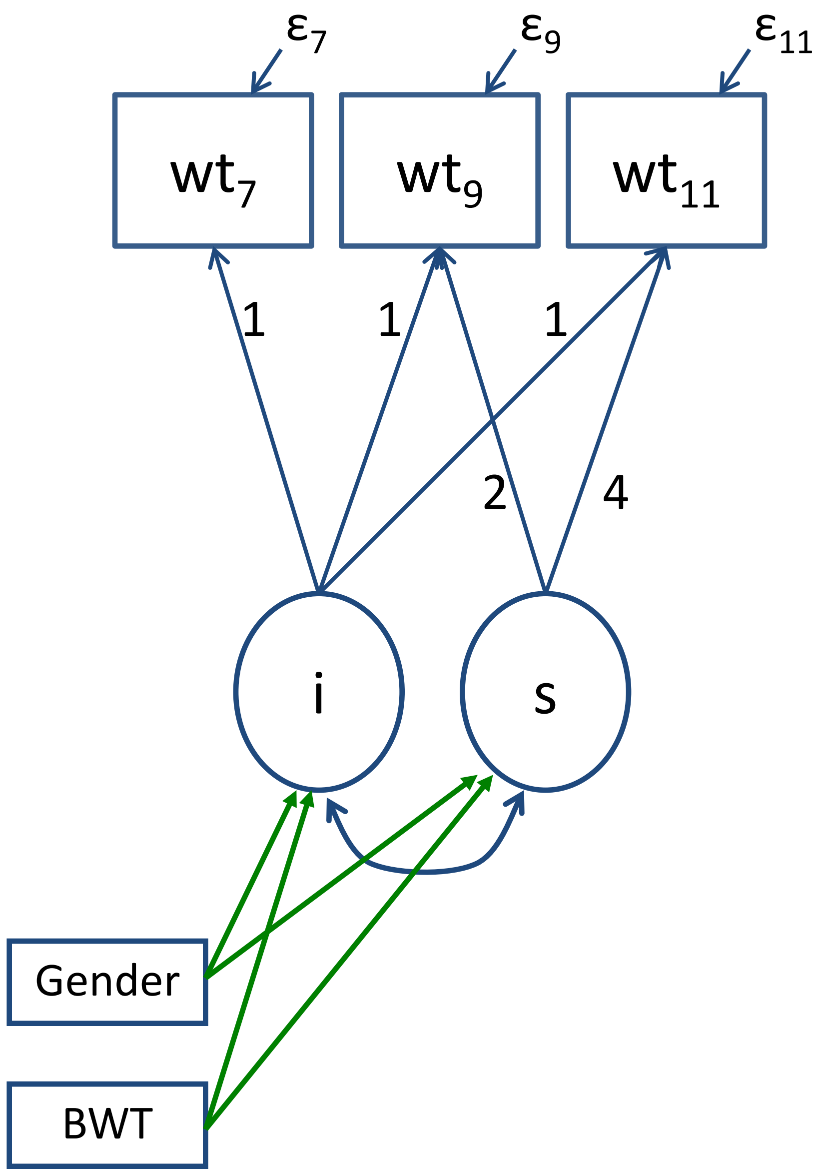
Notice large reduction in error variance

Adding Covariates

What explains variation in slope/intercept?

Adding covariates

- One aim with these models is to derive a measure or two that summarizes the growth which you can then use as an outcome
- Growth factors become just another variable that you can use as you would an observed measure (outcome/predictor/confounder/ mediator….)
- Model for gender is just a bivariate t-test innit?



Random I/S plus covariates (syntax)

```
Data:
  File is "R:\Research\users\majeh\Courses\Internal_mplus_3\repeatgrowth1.dta.dat" ;

Define:
    bwt = bwt/1000;

Variable:
  Names are id sex bwt
    age_07 ht_07 wt_07 bmi_07 age_09 ht_09 wt_09 bmi_09
    age_11 ht_11 wt_11 bmi_11 age_13 ht_13 wt_13 bmi_13
    age_15 ht_15 wt_15 bmi_15;
  Missing are all (-9999) ;
  usevariables = wt_07 wt_09 wt_11 bwt sex;

model:
   i by wt_7@1 wt_9@1 wt_11@1;
   s by wt_7@0 wt_9@2 wt_11@4;
   [wt_7@0 wt_9@0 wt_11@0];
   [i s];
   wt_7 wt_9 wt_11 (1);
   i s;
   i with s;
   i s on bwt sex;
```

Good idea to change the scale of BWT so estimates are more sensible – we can use the define command to do this

Our growth factors are regressed on birthweight (cts) and sex (binary)

Random I/S plus covariates (results)

```
MODEL RESULTS                                             Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
<snip>

 I        ON
    BWT                1.992      0.124     16.021      0.000
    SEX                0.001      0.137      0.006      0.995

 S        ON
    BWT                0.297      0.046      6.501      0.000
    SEX                0.450      0.050      8.955      0.000

 S        WITH
    I                  4.779      0.126     38.062      0.000

 Intercepts
    WT_07              0.000      0.000    999.000    999.000
    WT_09              0.000      0.000    999.000    999.000
    WT_11              0.000      0.000    999.000    999.000
    I                 18.676      0.491     38.040      0.000
    S                  2.726      0.180     15.125      0.000

 Residual Variances
    WT_07              3.068      0.070     44.061      0.000
    WT_09              3.068      0.070     44.061      0.000
    WT_11              3.068      0.070     44.061      0.000
    I                 15.501      0.414     37.450      0.000
    S                  2.050      0.056     36.662      0.000
```

Intercept related to birthweight
Slope related to birthweight and gender

Instead of factor means and variances
We have factor intercepts and residual variables
(the factors are now outcomes)

Mis-specified model results:-

	I on bwt	I on sex	S on bwt	S on sex
Random intercept / random slope	**1.992** (0.12)	**0.001** (0.14)	**0.297** (0.05)	**0.450** (0.05)
Random intercept / fixed slope	**2.586** (0.19)	**0.901** (0.21)	-	-
Random intercept / no slope	**2.586** (0.19)	**0.901** (0.21)	-	-

Failure to allow slopes to vary forces other aspects of the measurement model to compensate

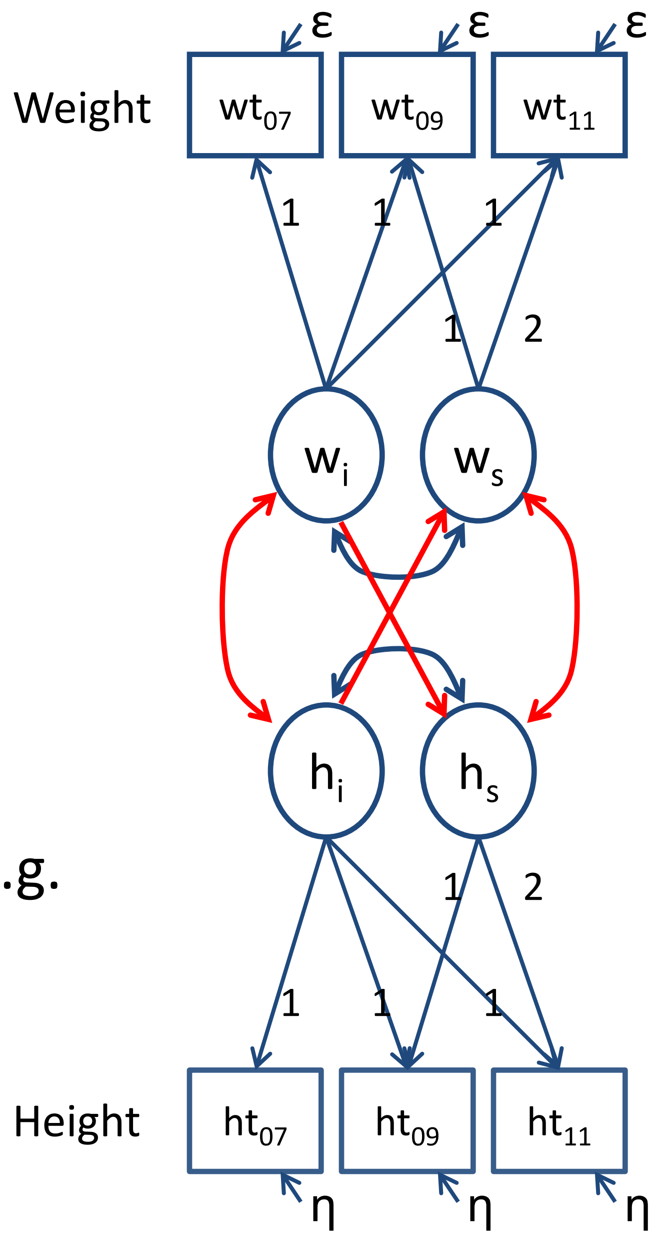
Consequently we can make incorrect conclusions about effect of gender

Model extension

Parallel models for height and weight

Parallel model of weight and height

- In addition to estimating
  - Mean and variance of I/S
  - Covariance between I and S
- We can examine how growth factors covary between the two processes, e.g.
  - cov(w_i, h_i), cov(w_s, h_i) …
- We could even regress w_s on h_i



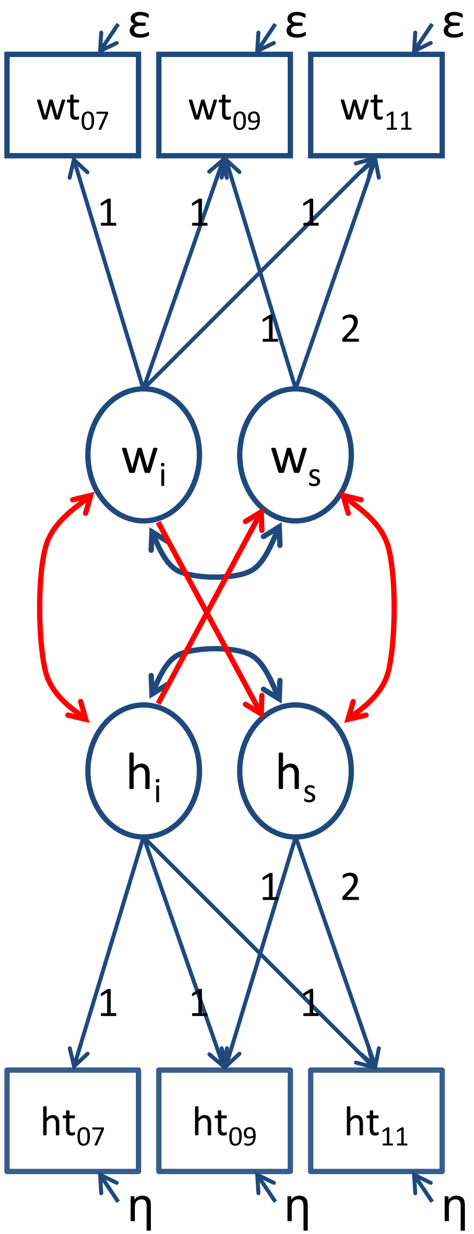
What about degrees of freedom?

- Don't panic, we have plenty now!

- 6 repeated measures means

  6+5+4+3+2 = 15 degrees of freedom
- i.e. more than if you modelled the two processes separately

Let's sweep empirical identification under the carpet for now



Parallel model of weight and height

```
Variable:
  Names are id sex bwt
     age_07 ht_07 wt_07 bmi_07 age_09 ht_09 wt_09 bmi_09
     age_11 ht_11 wt_11 bmi_11 age_13 ht_13 wt_13 bmi_13
     age_15 ht_15 wt_15 bmi_15;
  Missing are all (-9999) ;
  usevariables = wt_07 wt_09 wt_11 ht_07 ht_09 ht_11;

model:
    wi ws | wt_07@0 wt_09@2 wt_11@4;
    wt_07 (1);
    wt_09 (1);
    wt_11 (1);
    hi hs | ht_07@0 ht_09@2 ht_11@4;
    ht_07 (2);
    ht_09 (2);
    ht_11 (2);

    hi with wi ws hs;
    wi with ws hs;
    ws with hs;
```

Parallel model of weight and height

```
MODEL RESULTS                                               Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

  HI       WITH
    WI                16.655      0.464     35.884      0.000
    WS                 3.211      0.146     21.959      0.000
    HS                 1.874      0.085     22.101      0.000

 WI       WITH
    WS                 4.940      0.131     37.651      0.000
    HS                 0.821      0.067     12.217      0.000

 WS  WITH  HS          0.828      0.027     30.406      0.000

 Means
    WI                25.480      0.070    361.797      0.000
    WS                 4.421      0.025    174.049      0.000
    HI               125.969      0.086   1456.991      0.000
    HS                 6.297      0.015    419.192      0.000

 Variances
    WI                16.703      0.441     37.879      0.000
    WS                 2.121      0.058     36.886      0.000
    HI                26.202      0.662     39.589      0.000
    HS                 0.453      0.022     20.494      0.000

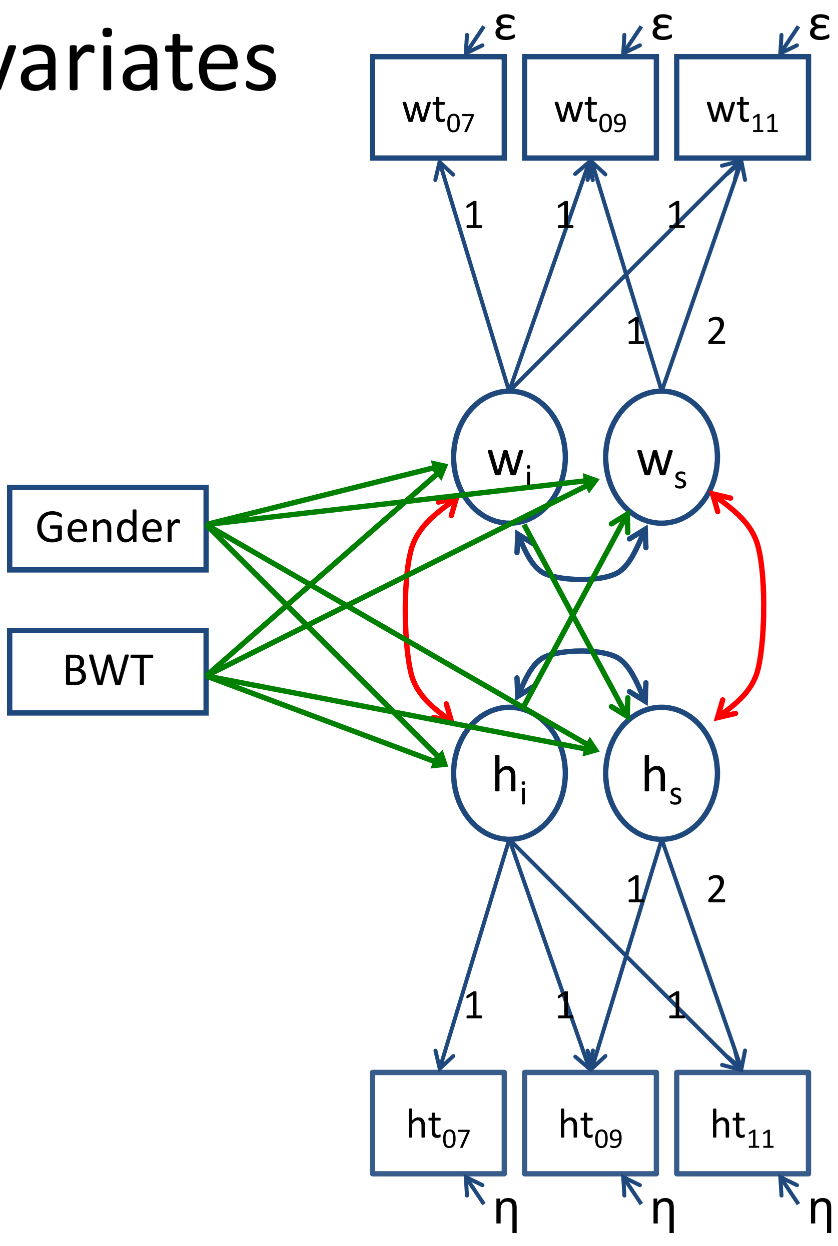
 Residual Variances
    WT_*               3.068      0.070     44.062      0.000
    HT_*               3.388      0.077     44.062      0.000
```

Adding covariates

```
model:
    wi ws | wt_07@0 wt_09@2 wt_11@4;
    wt_07 (1);
    wt_09 (1);
    wt_11 (1);
    hi hs | ht_07@0 ht_09@2 ht_11@4;
    ht_07 (2);
    ht_09 (2);
    ht_11 (2);

    hi with wi;
    hs with ws;
    hi with hs;
    wi with ws;

    hi on sex bwt;
    wi on sex bwt;
    hs on sex bwt wi;
    ws on sex bwt hi;
```



Adding covariates - results

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
HI ON				
SEX	-1.051	0.167	-6.289	0.000
BWT	2.368	0.152	15.593	0.000
WI ON				
SEX	**0.001**	**0.137**	**0.007**	**0.995**
BWT	1.992	0.124	16.023	0.000
HS ON				
SEX	0.562	0.028	19.992	0.000
BWT	-0.077	0.027	-2.904	0.004
WI	0.053	0.004	13.870	0.000
WS ON				
SEX	0.586	0.047	12.563	0.000
BWT	**-0.011**	**0.044**	**-0.242**	**0.808**
HI	0.130	0.005	26.007	0.000
HI WITH WI	15.184	0.430	35.330	0.000
HI WITH HS	1.218	0.057	21.225	0.000
HS WITH WS	0.353	0.023	15.464	0.000
WI WITH WS	2.808	0.082	34.055	0.000