Modelling executive function test performance in children

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Collaborative work

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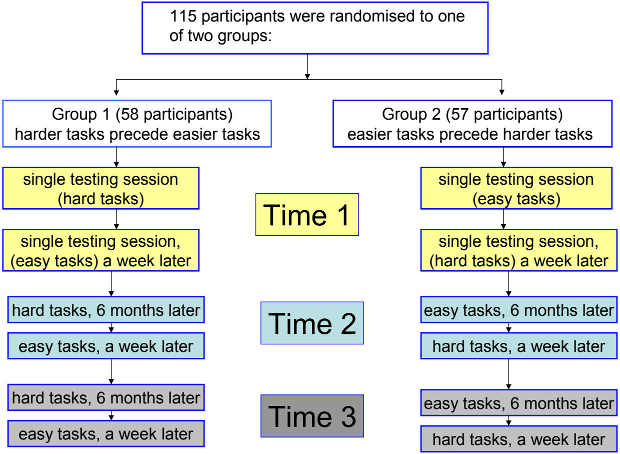
Background

- What is executive function?
- Components: inhibitory control, attentional flexibility, working memory, planning
- Competing models
- Experiments conducted by Shimmon (2004)

Executive tests used

Component	Measure	Version 1	Version 2
Inhibitory control	"Stroop"	Day/night	Abstract pattern
Attentional flexibility	Card-sort (DCCS)	Face-down	Face-up
Working memory	Boxes tasks	Scrambled	Stationary
	Digit-span (Times 2 & 3)	Backward	Forward
Planning	Tower of London	Subgoal Tower/Mixed	No-subgoal Tower/Mixed

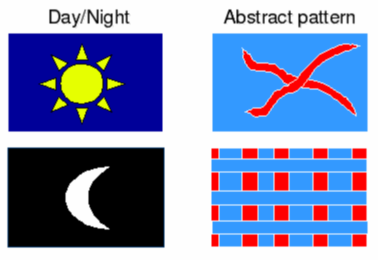
A longitudinal study



Aims of the study

1. Methodological questions concerning each executive function. For example,
   - identify patterns on the dynamics of test performance, within single sessions and over time periods;
   - evaluate the influence of one test upon another.
2. Relationships between executive functions

Inhibitory control



- abstract pattern (control)
- 16 trials at each session a week apart
- 3 sessions 6 months apart

Methodological questions

- Analyse key changes in the dynamics of test performance.
- Compare performance between abstract pattern and day/night tests.
- Evaluate the influence of one test upon another.
- Identify factors that influence performance e.g. age.

Modelling approach of IC data

- We assume the existence of an unobservable underlying ability, for each child. We represent such unobservable ability by a subject specific effect.
- Conditional on the subject specific effect we specify a dynamic model (Aalen et al, 2004) for each series of dependent outcomes.
- We extend the model to include the effect of time between test sessions.

Model specification: Part I

conditioning on the past and subject specific effects

$$Y_{ijk} \quad i = 1, \ldots, 32 \qquad j = 1, \ldots, 115 \qquad k = 1, 2, 3$$

$$\pi_{ijk} \equiv \Pr\left\{ Y_{ijk} = 1 | Y_{i-1,j,k}, S_{ijk}, \boldsymbol{X}_{jk}, \boldsymbol{Z}_{ijk}, U_j; \boldsymbol{\phi} \right\}$$

$$\begin{aligned} \operatorname{logit}\left(\pi_{ijk}\right) &= \log\left(\frac{\pi_{ijk}}{1 + \pi_{ijk}}\right) \\ &= \boldsymbol{X}'_{jk}\boldsymbol{\beta} + \boldsymbol{Z}'_{ijk}\boldsymbol{\delta} + \gamma_1 Y_{i-1,j,k} + \gamma_2 S_{ijk} + U_j \end{aligned}$$

We assume the U_j's to be an independent random sample from a normal distribution.

Model specification. Part II

Two ways of looking at longitudinal change

(i) specify different sets of regression parameters at each time period

$$\text{logit}\left(\pi_{ijk}\right) = \boldsymbol{X}'_{jk}\boldsymbol{\beta}_k + \boldsymbol{Z}'_{ijk}\boldsymbol{\delta}_k + \gamma_{1k} Y_{i-1,j,k} + \gamma_{2k} S_{ijk} + U_{jk}$$

(ii) consider common regression parameters at three time periods and a period effect

$$\text{logit}\left(\pi_{ijk}\right) = \boldsymbol{X}'_{jk}\boldsymbol{\beta} + \boldsymbol{Z}'_{ijk}\boldsymbol{\delta} + \gamma_1 Y_{i-1,j,k} + \gamma_2 S_{ijk} + \eta_k + U_j$$

Likelihood factorisation

Notation

We omit the index k without loss of generality.
Let $\boldsymbol{\phi} = (\boldsymbol{\beta}, \boldsymbol{\delta}, \gamma_1, \gamma_2, \eta)'$
and $\boldsymbol{W}_{ij} = (\boldsymbol{X}_j, \boldsymbol{Z}_{ij}, Y_{i-1,j}, S_{ij})'$.

Thus

$$\eta_{ij} = \text{logit}(\pi_{ij}) = \boldsymbol{W}'_{ij}\boldsymbol{\phi} + U_j$$

- Vector $\boldsymbol{\phi}$ contains the parameters of primary interest, and
- U_j's are regarded as nuisance parameters.

Likelihood factorisation (cont.)

The likelihood function is proportional to

$$\prod_j \frac{exp\left(\sum_i y_{ij}\eta_{ij} + U_j t_j\right)}{\prod_i \{1 + exp(\eta_{ij} + U_j)\}},$$

where $t_j = \sum_i y_{ij}$, and can be expressed as:

$$\prod_j \sum_L \frac{exp\left\{\sum_l y_{lj}\eta_{ij} + U_j t_j\right\}}{\prod_i \{1 + exp(\eta_{ij} + U_j)\}} \prod_j \frac{exp\left\{\sum_i y_{ij}\eta_{ij}\right\}}{\sum_L exp\left\{\sum_l y_{lj}\eta_{ij}\right\}}$$

$$= \prod_j L_M(\boldsymbol{\phi}, U_j; t_j) \prod_j L_C(\boldsymbol{\phi}; y_{ij}|t_j)$$

Statistical inference

- Statistical inference for $\boldsymbol{\phi}$ based on $\prod_j L_C(\boldsymbol{\phi}; y_{ij}|t_j)$ above is suitable because it does not make distributional assumptions about the subject-specific effects; however
- regression coefficients of covariates that do not change within cluster are non-identifiable.
- Therefore we adopt a random effects model, but
- we compare our results for the identifiable parameters.

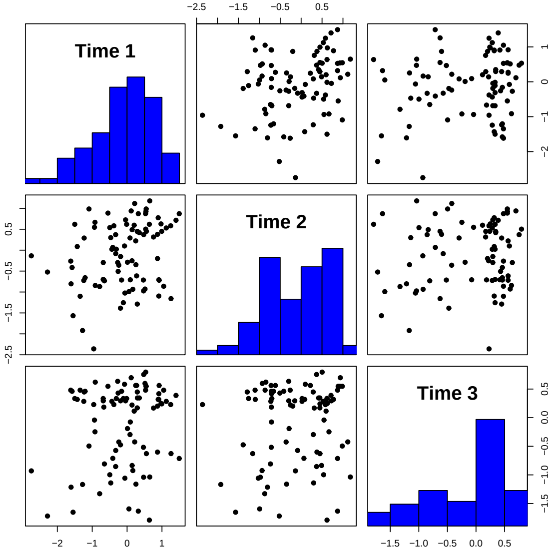
Results

Different sets of regression parameters at each time period

Table: MLE of parameters from random effects model (i)

Para-meters	Time 1		Time 2		Time 3	
	Est.	SE	Est.	SE	Est.	SE
Age β	0.13	0.021	0.13	0.030	0.12	0.040
Test δ_1	-1.29	0.13	-1.35	0.20	-1.066	0.31
Gp δ_2	0.48	0.25	0.75	0.38	-0.018	0.51
T$\times$gp δ_{12}	0.42	0.19	-0.073	0.30	0.25	0.41
Pr. ob. η_1	1.32	0.10	2.054	0.16	2.44	0.24
S. ord. η_2	-0.044	0.011	-0.056	0.016	-0.080	0.024
-2LogL:	-5672.1					

Estimated posterior modes of random effects



Separating between– and within–effects of covariates

- Neuhaus (2006) suggests to separate the effects of covariates in generalised linear mixed effects models in order to avoid a potential model misspecification.
- Note that separation of covariates into within- $(W_{ij} - \bar{W}_j)$ and between- $(\bar{W}_j)$ components in the conditional likelihood L_C yields:

$$\frac{exp\left\{\sum_i y_{ij}(W_{ij} - \bar{W}_j)'\eta_{ij}\right\}}{\sum_L exp\left\{\sum_l y_{lj}(W_{ij} - \bar{W}_j)'\eta_{ij}\right\}}$$

- Thus the conditional approach only estimates within-components of covariates effects.

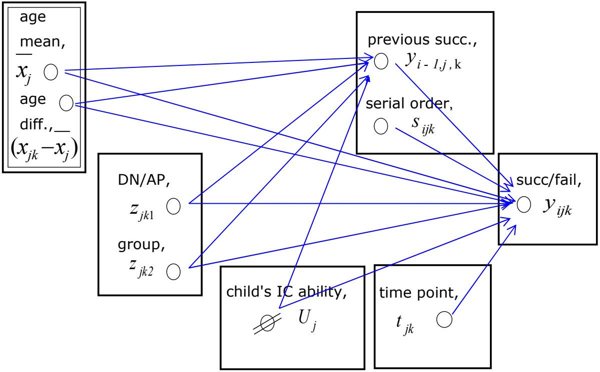
Results: Separating between– and within–cluster age effect

Common regression parameters at three time periods

Table: MLE of parameters from a random effects model (ii)

Parameters	Model 1		Model 2	
	Estimate	SE	Estimate	SE
Age β	0.12	0.016		
Age mean $\beta_{\mathbf{B}}$			0.11	0.016
Age dif. $\beta_{\mathbf{W}}$			0.18	0.037
Test (DN vs. AP) δ_1	-1.083	0.098	-1.082	0.098
Group (2 vs. 1) δ_2	0.35	0.19	0.38	0.19
Prev. obs. η_1	2.05	0.075	2.05	0.075
Serial order η_2	-0.043	0.008	-0.043	0.008
Time (2 vs. 1) γ_2	0.34	0.12	-0.031	0.23
Time (3 vs. 1) γ_3	0.52	0.21	-0.23	0.44
Test$\times$group δ_{12}	0.21	0.14	0.21	0.14
-2Log-likelihood:	-5892.5		-5888.74	

Graphical representation of results

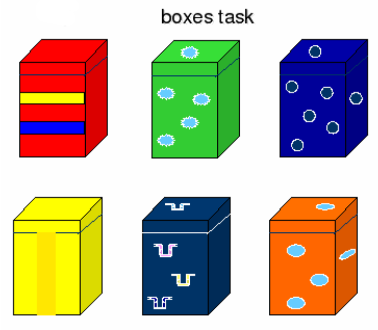


Results in words

Maximum likelihood estimates of regression coefficients suggest:

1. A fatigue effect in the performance of a given child, as indicated by the negative effect associated to trial index ($\hat{\eta}_2 = -0.043$, se$(\hat{\eta}_2) = 0.008$). In contrast,
2. a success in the previous trial increases the chances of success in subsequent trials ($\hat{\eta}_1 = 2.05$, se$(\hat{\eta}_2) = 0.075$).
3. Children perform better at the AP task than at the DN task ($\hat{\delta}_1 = -1.082$, se$(\hat{\delta}_1) = 0.098$) .
4. Children who took AP before DN task performed better than those who took the test in the reverse order $\hat{\delta}_2 = 0.38$, se$(\hat{\delta}_2) = 0.19$),

Working memory



- stationary vs. scramble boxes
- sequences of succ/fail until retrieving 6 sweets
- 3 sessions 6 months apart

Boxes tasks data

- Let $\boldsymbol{Z}_{jk} = (z_{ijk}, \ldots, z_{n_j jk})$ fail/succ to retrieve a sweet in n_j trials at time period k
- Let $S_{ijk} = 5 - \sum_{l=1}^{i} z_{ljk}$ No. of sweets that remain to be retrieved at trial i^{th} and time period k.
- We model $P_{ijk} = Pr(z_{ijk} = 1 | s_{ijk} = s)$, for $s = 1, \ldots, 5$ as

$$\text{logit}(P_{ijk}) = \alpha_s + \boldsymbol{X}'_{ijk}\boldsymbol{\beta}_k + \gamma_k + U_j$$

Statistical inference

The parameters of primary interest are the regression parameters and the subject-specific effects are regarded as nuisance parameters. Recall that $S_{ijk} = 5 - \sum_{l=1}^{i} z_{ljk}$. The likelihood function is:

$$L(\alpha_s, \boldsymbol{\beta}; \boldsymbol{Z}_{jk}) = \prod_{jk} \int \prod_{s \geq 1} \left\{ \left[\prod_{failures} \left(1 - P_{ijk}\right) \right] P_{ijk} \right\} f(U_j; \theta) \mathrm{d}U_j,$$

where $f(U_j; \theta)$ is the density function of the latent variable U_j

Statistical inference (cont.)

- As with the inhibitory control data we adopt a random–effects model, but we also compare results with a conditional likelihood approach.
- Similarly we investigate for within– and between–effects of age.

Table: MLE of parameters for boxes tasks data from a random effects model

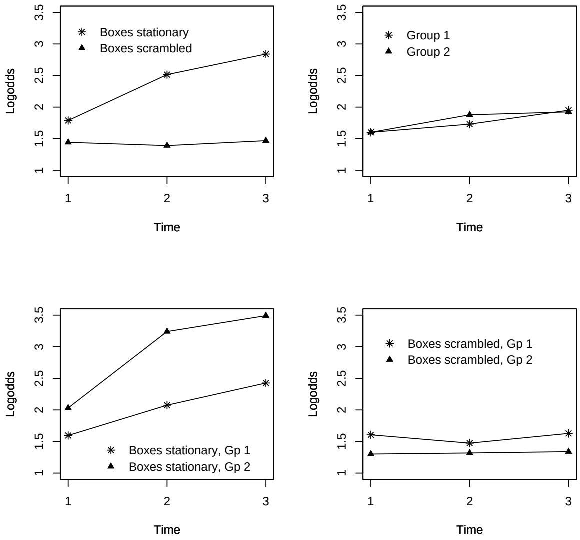
Parameters	Estimate	SE
Age mean $\beta_{\mathbf{B}}$	0.058	0.018
Age dif. $\beta_{\mathbf{W}}$	0.044	0.049
Test (Scr vs. Sta) δ_1	-0.014	0.19
Group (2 vs. 1) δ_2	0.56	0.25
Time (2 vs. 1) γ_2	0.41	0.35
Time (3 vs. 1) γ_3	0.43	0.60
Test*group δ_{12}	-0.93	0.22
Time (2 vs. 1)*Test $\gamma_2\delta_1$	-0.75	0.28
Time (3 vs. 1)*Test $\gamma_3\delta_1$	-1.017	0.30
-2Log-likelihood:-2170		

Recall...

Test order at each time point

Week 1		Week 2	
Group 1:	harder tasks	Group 1:	easier tasks
Group 2:	easier tasks	Group 2:	harder tasks

Plots of overall logodds for boxes tasks data



Impurity of boxes tests

- strong effect of order (of a different nature to that of IC tests)
- children who took the easy test version first, performed better at the stationary but not at scramble version

Concluding remarks

- We investigated how succ/fail in previous trials affect future performance,
- aggregates of succ/fail will loose information on the dynamics of the sequence.
- There is value in separating practice effects from age effects.
- Finally, we emphasize that testing order should not be ignored.

References

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