

# Social Choice and Individual Reports of Subjective Well-Being

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ESRC Research Methods Festival



#### Session Title: Happiness, Income and Welfare

In the face of evidence that individuals may lack coherent and/or stable preferences, is there a case for using happiness data to underpin economic policy?

Typically, large scale survey (panel) data on individual's self-reported subjective well-being (SWB) is regressed on a number of independent variables of interest.

A number of such studies have provided evidence that well-being self-reports are correlated with objective measures such as health measures.

However, the use of such data on its own as a normative underpinning for policy raises a number of concerns that will be addressed in this session. ESRC Research Methods Festival St. Caths. Oxford. 2012 July 5th 2/ 55

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- Four Income Distributions
- Dominance

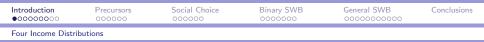


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#### Income Data

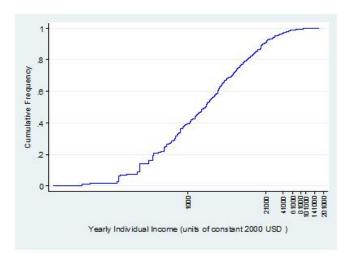
Accessible data source: *World Values Survey* (WVS), five waves between 1981 and 2007, total of 117,876 observations.

Yearly Individual Income measured in year 2000 US dollars:

- extract the lower and upper bounds of the income range reported by the interviewee;
- transform it to annual income;
- use an interval regression to estimate a probability distribution of possible incomes for each interviewee;
- use data from World Development Indicators (WDI) 2010 to correct for exchange rates and price changes.

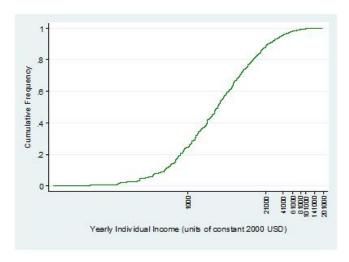
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Four Income Distributions							

#### Income Distribution: Blue



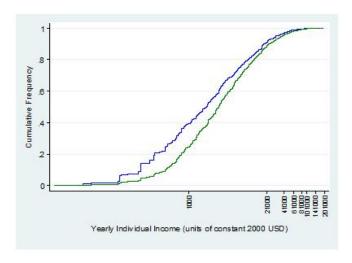
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## Income Distribution: Green



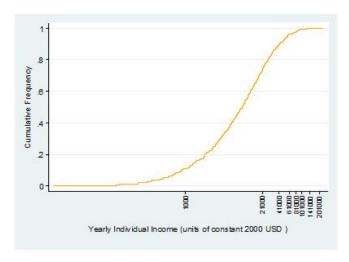
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#### Income Distribution: Comparing Blue and Green



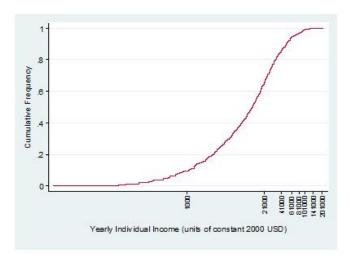
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#### Income Distribution: Yellow



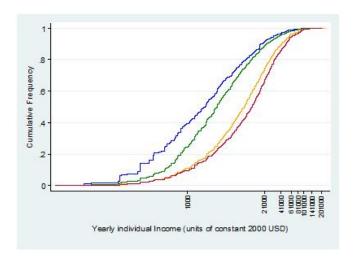
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## Income Distribution: Red



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#### Four Income Distributions Together



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## Income Correlates

What are these colours that lie behind these (stochastically dominating) rightward shifts in the conditional income distribution?

The *World Values Survey* also has people report a level of happiness:

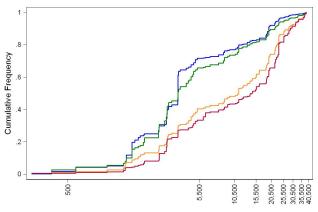
- blue for "not at all happy" (2.55%);
- 2 green for "not very happy" (15.16%);
- **3** yellow for "quite happy" (53.94%);
- red for "very happy" (28.35%).

Income distribution conditional on a higher happiness level stochastically dominates

income distribution conditional on a lower happiness level.

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## National GDP



Yearly Per Capita GDP (units of constant 2000 USD)

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#### **5** General SWB

#### 6 Conclusions

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## **Psychological Precursors**

Watson, Goodwin (1930)

"Happiness Among Adult Students of Education"

Journal of Educational Psychology 21: 79–109.

Wilson, Warner R. (1967) "Correlates of Avowed Happiness" *Psychological Bulletin* 67: 294–306.

Diener, Ed (1984) "Subjective Well-Being" *Psychological Bulletin* 95: 542–575.

Diener, Ed, Eunkook Suh, Richard Lucas, and Heidi Smith (1999) "Subjective Well-Being: Three Decades of Progress" *Psychological Bulletin*, 125: 276–302.



## Easterlin paradox

Easterlin, Richard A. (1974) "Does Economic Growth Improve the Human Lot? Some Empirical Evidence" in Paul A. David and Melvin W. Reder (eds.) *Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz* (New York: Academic Press) Study of the US during the period between 1946 and 1970. Economic growth appeared not to have enhanced SWB. Though US income per person rose steadily,

average reported happiness showed no long-term trend, and actually declined between 1960 and 1970.

Contrast between the views of two late Stanford colleagues:

- Moses Abramovitz, historian of economic growth
- Tibor Scitovsky, *The Joyless Economy*

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### Easterlin paradox: Is it relevant?

Within a given country, people with higher incomes are more likely to report being happy.

The joint worldwide distribution indicates that happiness and income are also positively associated, suggesting that Easterlin's findings have limited extent.

But the average reported level of happiness within a country varies much less w.r.t. national income per person, at least for countries with income sufficient to meet basic needs.

The happiness reports indicate something statistically significant. But we can still dispute what is the significance.

Or even the hypothesis that income causes happiness, as opposed to happiness causing income.

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First Normative Extreme: "(Neo-)classicism"

Abramovitz(?): Social choice theory and welfare economics should ignore SWB measures altogether, treating them as mere noise.

Two claims:

Positive Income growth and wealth accumulation are inherently desirable.

Strong Negative Empirical analysis of SWB has no normative significance.

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## Kuznets' Scepticism

Simon Kuznets, whom President Hoover had commissioned to conduct the surveys needed to measure national income, and eventually won a Nobel Prize in Economics for his work, reported in 1934 that: "The welfare of a nation can scarcely be inferred from a measure of national income."

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# Second Normative Extreme: "Hedonism"

Scitovsky(?): Some propose welfare measures based exclusively on SWB.

A BBC survey in 2006 concluded that 81% of Britain's population would rather the government make them happier than richer. Under this approach, Easterlin's paradox might suggest we should not even try to promote growth or development. Francis Ysidro Edgeworth (1881) *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* Possibilities for compromise will be explored.

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## Layard, Mayraz, and Nickell: Descriptive Hypothesis

Layard, Richard, Guy Mayraz, and Stephen Nickell (2008) "The Marginal Utility of Income" *Journal of Public Economics* 92: 1846–1857.

They estimate the equation  $h = u(y) + \mathbf{b}^{\top} \mathbf{x} + \epsilon$  with  $\mathbb{E}\epsilon = 0$ . Here *h* is SWB, *y* is income, and **x** is a vector of other observable individual characteristics that may be relevant to both:

- individuals' reports of their own SWB;
- **2** social value judgements of individual welfare.

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# Psychological Inequality Aversion

Moreover, they take  $d \ln u/d \ln y = -a$ , where the parameter a is the the Atkinson measure of (constant) relative inequality aversion. To be compared with the Arrow–Pratt measure of relative risk aversion.

They call  $u'(y) = y^{-a}$  the "marginal utility" of income, though it is merely marginal happiness or, at best, psychological marginal utility.

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## Layard, Mayraz, Nickell: Prescriptive Hypothesis

The function  $u'(y) = y^{-a}$  need not be the same as the ethical marginal utility of income which is important for cost-benefit analysis and optimal taxation. For it to be so involves an extra "Benthamite" prescriptive hypothesis. Given an income distribution  $y^N$  and a profile  $\mathbf{x}^N$ of other observable individual characteristics relevant to welfare, it is assumed that social welfare can be measured by the sum

$$W(y^N, \mathbf{x}^N) \equiv \sum_{i \in N} \mathbb{E}[h_i | y_i, \mathbf{x}_i] \equiv \sum_{i \in N} [u(y_i) + \mathbf{b}^\top \mathbf{x}_i]$$

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#### Individuals and Consequences

Consider a society made up of a finite set N of individuals.

Consequences are individualized social states  $z^N \in Z^N := \prod_{i \in N} Z_i$ , where each  $z_i \in Z_i$  is a personal consequence for individual *i*.

A social consequence lottery  $\lambda \in \Delta(Z^N)$  is a finitely supported probability distribution on the consequence space.

For each  $i \in N$  and  $\lambda \in \Delta(Z^N)$ , let  $\lambda_i \in \Delta(Z_i)$  denote the induced marginal distribution on *i*'s personal consequences.

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## Decisions and NM Utilities

Two von Neumann–Morgenstern utility functions (or NMUFs)  $w, \tilde{w} : Z^N \to \mathbb{R}$  are cardinally equivalent just in case there exist an additive constant  $\alpha$ and a multiplicative constant  $\rho > 0$ such that  $\tilde{w}(z^N) \equiv \alpha + \rho w(z^N)$ .

Rationality of normative behaviour in ethical decision trees implies there exists a unique cardinal equivalence class of NMUFs

$$Z^N \ni z^N \mapsto w(z^N) \in \mathbb{R}$$

whose expected values  $\mathbb{E}_{\lambda}[w(z^{N})]$ are all maximized simultaneously w.r.t.  $\lambda$ .

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# Individualistic Consequentialism

#### Assumption

If the two social consequence lotteries  $\lambda, \mu \in \Delta(Z^N)$  induce equal marginal personal lotteries  $\lambda_i = \mu_i \in \Delta(Z_i)$  for all  $i \in N$ , then  $\lambda$  and  $\mu$  are socially equivalent.

In particular, they are socially indifferent, so the respective expected values of the social NMUF wmust satisfy  $\mathbb{E}_{\lambda}w(z^N) = \mathbb{E}_{\mu}w(z^N)$ .

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## Individual Welfarism

#### Assumption

For each individual  $i \in N$ , there is a unique cardinal equivalence class of personal NMUFs  $z_i \mapsto w_i(z_i)$  which, for each fixed profile  $\overline{z}^{N \setminus \{i\}} \in \prod_{j \in N \setminus \{i\}} Z_j$ of personal consequences  $z_j \in Z_j$  for individuals  $j \neq i$ , are all cardinally equivalent to  $z_i \mapsto w(z_i, \overline{z}^{N \setminus \{i\}})$ .

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# Additive Utilitarianism

**Definition**: Two families  $\langle w_i \rangle_{i \in N}$ ,  $\langle \tilde{w}_i \rangle_{i \in N}$  of personal NMUFs are co-cardinally equivalent just in case there exist a family  $\langle \alpha_i \rangle_{i \in N}$  of additive constants and a single multiplicative constant  $\rho > 0$ , independent of *i*, such that  $\tilde{w}_i(z^N) \equiv \alpha_i + \rho w_i(z^N)$ .

**Theorem**: Together, individualistic consequentialism and individual welfarism imply that, except in trivial cases, there exists a unique co-cardinal equivalence class of interpersonally comparable NMUFs  $N \times Z \ni (i, z) \mapsto w_i(z) \in \mathbb{R}$ such that the social NMUF  $z^N \mapsto w(z^N)$ can be written in the additive utilitarian form

$$z^N \mapsto w(z^N) = \sum_{i \in N} w_i(z_i).$$

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#### Individual Welfare Types

**Assumption**: Each individual  $i \in N$  also has a fixed type  $t_i \in T$ . Let  $t^N \in T^N := \prod_{i \in N} T_i$  denote the fixed "welfare" type profile. This type profile  $t^N$ , together with a "master" utility function

$$Z \times T \ni (z, t) \mapsto v(z; t)$$

satisfying  $w_i(z_i) \equiv v(z_i; t_i)$  for all  $i \in N$  and  $z_i \in Z$ , is assumed to determine the co-cardinal equivalence class

$$N \times Z \times T^N \ni (i, z, t^N) \mapsto v(z; t_i),$$

and so the social NMUF

$$z^N \mapsto w(z^N; t^N) = \sum_{i \in N} v(z_i; t_i).$$

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- Dominance Conditions

#### **5** General SWB

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# **Binomial Choice Model**

Suppose SWB is measured by a binary variable  $s \in \{0, 1\}$ , where:

- s = 1 indicates general life satisfaction;
- s = 0 indicates general life dissatisfaction.

Assume we can observe the function

$$Z \times T \ni (z,t) \mapsto p(z,t) \in [0,1]$$

whose value is the proportion of type t individuals who face personal consequence  $z \in Z$  and report s = 1.

This makes p(z, t) more informative than s itself.

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# Ordinal Non-Comparable Welfare

In the Arrow (1951) ordinal non-comparable (ONC) case, one could make the ethical value judgement that for each fixed type  $t \in T$ , the two functions  $z \mapsto p(z, t)$  and  $z \mapsto v(z; t)$  are ordinally equivalent on Z.

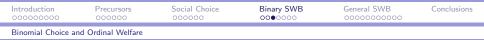
For each separate type  $t \in T$ , there must be a strictly increasing transformation  $\mathbb{R} \ni \xi \mapsto \psi_t(\xi) \in \mathbb{R}$  such that  $v(z; t) = \psi_t(p(z, t))$ .

It follows that the additive NMUF takes the form

$$w(z^N;t^N) = \sum_{i\in N} \psi_{t_i}(p(z_i,t_i))$$

From now on, let  $\Psi$  denote the class of all transformations  $\psi : T \times \mathbb{R} \to \mathbb{R}$ with the property that  $\xi \mapsto \psi_t(\xi)$ is strictly increasing for each welfare type  $t \in T$ . ESRC Research Methods Festival St. Caths, Oxford, 2012 July 5th

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## Ordinal Level Comparable Welfare

In the ordinal level comparable (OLC) case, one could make the ethical value judgement that the two functions  $(z, t) \mapsto p(z, t)$  and  $(z, t) \mapsto v(z; t)$ are ordinally equivalent on  $Z \times T$ , even as the type  $t \in T$  varies.

There must be *just one* strictly increasing transformation  $\mathbb{R} \ni \xi \mapsto \phi(\xi) \in \mathbb{R}$  such that  $v(z; t) = \phi(p(z, t))$ .

It follows that the additive NMUF takes the form

$$w(z^N;t^N) = \sum_{i\in N} \phi(p(z,t_i))$$

From now on, let  $\Phi$  denote the class of all strictly increasing transformations  $\phi : \mathbb{R} \to \mathbb{R}$ .

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# Pareto Dominance

**Proposition** Consider any fixed type profile  $t^N \in T^N$ . Suppose that  $z^N, \tilde{z}^N \in Z^N$ .

The following two statements are equivalent:

**ISP** (independent strict preference)

$$\sum_{i\in N}\psi_{t_i}(p(z_i,t_i))>\sum_{i\in N}\psi_{t_i}(p(\widetilde{z}_i,t_i))$$

for all  $\psi \in \Psi$  simultaneously;

PD (Pareto dominance)  $p(z_i, t_i) \ge p(\tilde{z}_i, t_i)$  for all  $i \in N$ , with strict inequality for at least one  $i \in N$ .

**Corollary** Without more information about  $(z, t) \mapsto v(z; t)$ , given any feasible alternative set  $A \subset Z^N$  and any  $z^N \in A$ :

*z<sup>N</sup>* is a possible social choice iff it is Pareto efficient; *z<sup>N</sup>* is an impossible social choice iff it is Pareto dominated.

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Suppes Dominance, I

**Definition** Given any fixed type profile  $t^N \in T^N$ , and any social consequence  $z^N \in Z^N$ ,

define the associated interpersonal cumulative distribution function

$$[0,1] 
i \xi \mapsto G(\xi;t^N,z^N) \in [0,1]$$

so that  $G(\xi; t^N, z^N)$  is the expected proportion of individuals whose personal consequence  $z_i$  induces a probability  $p(z_i, t_i)$ of expressing satisfaction that does not exceed  $\xi$ .

Note that a lower value of G for each  $\xi$  suggests an improvement in  $(t^N, z^N)$ .

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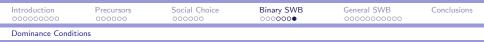
Suppes Dominance, II

Following Suppes (1966), given any welfare type profile  $t^N \in T^N$ and any fixed pair of social consequences  $z^N, \tilde{z}^N \in Z^N$ , say that  $z^N$  Suppes dominates  $\tilde{z}^N$ , and write  $z^N \succ_{t^N}^{SD} \tilde{z}^N$ , just in case the respective induced interpersonal CDFs satisfy:

$$\ \, {\bf G}(\xi;t^N,z^N)\leq G(\xi;t^N,\tilde{z}^N) \ \, {\rm for \ all} \ \, \xi\in[0,1];$$

 $\ \, {\it O} \ \, {\it G}(\xi;t^N,z^N) < {\it G}(\xi;t^N,\tilde{z}^N) \ \, {\it for at least one} \ \, \xi\in[0,1].$ 

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# Suppes Dominance III

**Proposition** Consider any fixed type profile  $t^N \in T^N$ . For any  $z^N, \tilde{z}^N \in Z^N$ , the following two statements are equivalent: ISP (independent strict preference)

$$\sum_{i\in N} \phi(p(z_i,t_i)) > \sum_{i\in N} \phi(p(\tilde{z}_i,t_i))$$

for all  $\phi \in \Phi$  simultaneously;

SD (Suppes dominance)  $z^N \succ_{t^N}^{SD} \tilde{z}^N$ .

**Corollary** Given any feasible set  $A \subset Z^N$  and any  $z^N \in A$ , without more information about the function  $(z, t) \mapsto v(z; t)$ :

*z<sup>N</sup>* is a possible social choice iff it is Suppes undominated;
 *z<sup>N</sup>* is an impossible social choice iff it is Suppes dominated.

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- Ordered Multinomial Choice
- Dominance Conditions

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# The Ordered Multinomial Choice Model

Suppose now that SWB ranges over the set  $S := \{0, 1, 2, \dots, \ell\}$ . The previous binary case had  $\ell = 1$ .

For each  $s \in S$ , let p(s|z, t) denote the conditional probability that an individual of type  $t \in T$  facing personal consequence  $z \in Z$ reports an SWB level s.

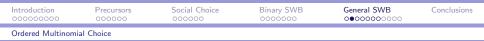
Consider the downwardly cumulated conditional probabilities

$$P(s|z,t) := \sum_{s'=s}^{\ell} p(s'|z,t)$$

that the individual reports an SWB level no lower than s.

Of course, in the binomial case when  $\ell = 1$ , one has P(0|z, t) = 1 and P(1|z, t) = p(z, t).

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# Stochastic Utility

**Definition**: For each type  $t \in T$ , the mapping  $Z \ni z \mapsto u_t(z)$  is a stochastic utility function if there exist both:

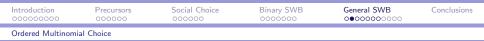
a strictly increasing and continuous CDF ℝ ∋ ξ → H<sub>t</sub>(ξ) whose range includes (0, 1);

• a strictly increasing sequence of constants  $(\xi_t^s)_{s=1}^{\ell}$ ;

with the property that  $P(s|z,t) = H_t(u_t(z) - \xi_t^s)$ for all SWB levels  $s \in S \setminus \{0\}$  and all consequences  $z \in Z$ .

It is conceptually entirely distinct from the random utility model: for each non-empty feasible subset  $F \subseteq Z$ , that model specifies the probability p(z, F) that an individual will choose each  $z \in F$ as equal to the probability that the random utility function gives z no less utility than the other members of F.

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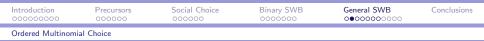
# Stochastic Utility

**Definition**: For each type  $t \in T$ , the mapping  $Z \ni z \mapsto u_t(z)$  is a stochastic utility function if there exist both:

- a strictly increasing and continuous CDF ℝ ∋ ξ → H<sub>t</sub>(ξ) whose range includes (0, 1);
- a strictly increasing sequence of constants  $(\xi_t^s)_{s=1}^{\ell}$ ; with the property that  $P(s|z,t) = H_t(u_t(z) - \xi_t^s)$ for all SWB levels  $s \in S \setminus \{0\}$  and all consequences  $z \in Z$ .

It is conceptually entirely distinct from the random utility model: for each non-empty feasible subset  $F \subseteq Z$ , that model specifies the probability p(z, F) that an individual will choose each  $z \in F$ as equal to the probability that the random utility function gives z no less utility than the other members of F.

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## Stochastic Utility

**Definition**: For each type  $t \in T$ , the mapping  $Z \ni z \mapsto u_t(z)$  is a stochastic utility function if there exist both:

a strictly increasing and continuous CDF ℝ ∋ ξ → H<sub>t</sub>(ξ) whose range includes (0, 1);

• a strictly increasing sequence of constants  $(\xi_t^s)_{s=1}^{\ell}$ ; with the property that  $P(s|z,t) = H_t(u_t(z) - \xi_t^s)$ for all SWB levels  $s \in S \setminus \{0\}$  and all consequences  $z \in Z$ .

It is conceptually entirely distinct from the random utility model: for each non-empty feasible subset  $F \subseteq Z$ , that model specifies the probability p(z, F) that an individual will choose each  $z \in F$ as equal to the probability that the random utility function gives z no less utility than the other members of F.

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Ordered Multinomial Choice						

## Ordered Logit and Probit

The interpersonally comparable stochastic utility function  $(z, t) \mapsto P(s|z, t) = H_t(u_t(z) - \xi_t^s)$ is consistent with standard ordered discrete choice models such as:

- ordered logit, where  $\ln[P/(1-P)] = \beta_t U$ , implying that  $P = H(U) = e^{\beta_t U}/(1+e^{\beta_t U})$ ;
- ordered probit, where  $P = H_t(U) = \Phi((U \mu_t)/\sigma_t)$ for the standard normal CDF  $\Phi(U) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^U e^{-\frac{1}{2}v^2} dv$ .



# Stochastic Utility and Welfare

**Proposition**: If type t has a stochastic utility function  $z \mapsto u_t(z)$ , then for all  $s \in S$  it is ordinally equivalent to  $z \mapsto P(s|z, t)$ .

**Proof**: Suppose that  $P(s|z,t) = H_t(u_t(z) - \xi_t^s)$ , where  $H_t$  is strictly increasing. Then

$$egin{aligned} \mathcal{P}(s|z,t) &\gtrless \mathcal{P}(s|z',t) &\iff \mathcal{H}_t(u_t(z)-\xi^s_t) &\gtrless \mathcal{H}_t(u_t(z')-\xi^s_t) \ &\iff u_t(z) &\gtrless u_t(z') \end{aligned}$$

By definition, therefore, for all  $s \in S$ , the two mappings  $z \mapsto P(s|z, t)$  and  $z \mapsto u_t(z)$ must be ordinally equivalent. QED

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# Ordinal Non-Comparable Welfare

In the ordinal non-comparable (ONC) case, one can judge that for each fixed type  $t \in T$ , the two functions  $z \mapsto u_t(z)$  and  $z \mapsto v(z; t)$ are ordinally equivalent on Z.

Provided ordinal equivalence holds, for each type  $t \in T$  and  $s \in S$ , there must be a strictly increasing transformation  $\xi \mapsto \psi_t^s(\xi)$  on  $\mathbb{R}$  such that  $v(z; t) = \psi_t^s(P(s|z, t))$ .

It follows that the additive NMUF takes the form

$$w(z^N; t^N) = \sum_{i \in N} \psi^s_{t_i}(u_{t_i}(z_i))$$

# Interpersonally Comparable Stochastic Utility

**Definition**: The mapping  $Z \times T \ni (z, t) \mapsto u_t(z)$  is an interpersonally comparable stochastic utility function if there exist both:

- a strictly increasing and continuous CDF ℝ ∋ ξ → H(ξ) whose range includes (0, 1);
- a strictly increasing sequence of constants  $(\xi_t^s)_{s=1}^{\ell}$ ;

with the property that  $P(s|z, t) = H(u_t(z) - \xi^s)$ for all types  $t \in T$ , SWB levels  $s \in S \setminus \{0\}$ , and personal consequences  $z \in Z$ .

**Proposition**: If there is an interpersonally comparable stochastic utility function  $(z, t) \mapsto u_t(z)$ , then it is ordinally equivalent, for all  $s \in S$ , to  $(z, t) \mapsto P(s|z, t)$ .

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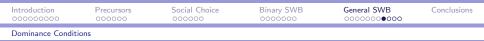
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Ordered Multinomial Choice							

## Ordinal Level Comparable Welfare

In the ordinal level comparable (OLC) case, one could judge that even as the type  $t \in T$  varies, the two functions  $(z, t) \mapsto P(s|z, t)$  and  $(z, t) \mapsto v(z; t)$ are ordinally equivalent on  $Z \times T$ .

It follows that the additive NMUF takes the form

$$w(z^N;t^N)=\sum_{i\in N}\phi(u_{t_i}(z_i))$$



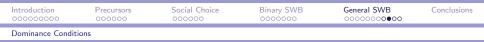
### Pareto Dominance

**Proposition** Consider any fixed type profile  $t^N \in T^N$ . Suppose that  $z^N, \tilde{z}^N \in Z^N$ . The following two statements are equivalent: ISP (independent strict preference)

$$\sum_{i\in\mathbb{N}}\psi_{t_i}^s(u_{t_i}(z_i))>\sum_{i\in\mathbb{N}}\psi_{t_i}^s(u_{t_i}(\tilde{z}_i))$$

for all  $s \in S$  and  $\psi^s \in \Psi$  simultaneously;

PD (Pareto dominance) for all  $s \in S$  simultaneously, one has  $P(s|z_i, t_i) \ge P(s|\tilde{z}_i, t_i)$  for all  $i \in N$ , with  $P(s|z_i, t_i) > P(s|\tilde{z}_i, t_i)$  for at least one  $i \in N$ .



### Interpersonal CDF

**Definition**: Given any fixed type profile  $t^N \in T^N$ , any SWB level  $s \in S$ , and any social consequence  $z^N \in Z^N$ , define the associated interpersonal cumulative distribution function

$$[0,1] 
i \xi \mapsto G^s(\xi;t^N,z^N) \in [0,1]$$

so that  $G^{s}(\xi; t^{N}, z^{N})$  is the expected proportion of individuals whose personal consequence  $z_{i}$  induces a probability  $P(s|z_{i}, t_{i})$ no higher than  $\xi$  of expressing an SWB level  $\geq s$ .

Recall that a lower value of G for each  $\xi$  suggests an improvement in  $(t^N, z^N)$ .

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### Suppes Dominance Revisited

**Proposition**: Consider any fixed type profile  $t^N \in T^N$ . For any  $z^N, \tilde{z}^N \in Z^N$ , the following two statements are equivalent: ISP (independent strict preference)

$$\sum_{i\in N}\phi(u_{t_i}(z_i))>\sum_{i\in N}\phi(u_{t_i}(\tilde{z}_i))$$

for all  $s \in S$  and  $\phi \in \Phi$  simultaneously;

SD (Suppes dominance) For all  $s \in S$  simultaneously, one has  $G^{s}(\xi; t^{N}, z^{N}) \leq G^{s}(\xi; t^{N}, \tilde{z}^{N})$  for all  $\xi \in [0, 1]$ , with  $G^{s}(\xi; t^{N}, z^{N}) < G^{s}(\xi; t^{N}, \tilde{z}^{N})$  for at least one  $\xi \in [0, 1]$ .

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### Layard, Mayraz and Nickell

LMN's regression equation  $s = u(y) + \mathbf{b}^{\top} \mathbf{x} + \epsilon$ can be recast as an interpersonally comparable stochastic utility function  $(z, t) \mapsto P(s|z, t) = H(u_t(z) - \xi^s)$ , with CDF  $\epsilon \mapsto H(\epsilon)$ , and with  $u_t(z) - \xi^s$  replaced by  $u(y) + \mathbf{b}^{\top} \mathbf{x} - s$ , independent of t.

This allows empirical data to determine the form of the psychological marginal utility function  $y \mapsto u'(y)$ and so of the stochastic utility function  $(y, \mathbf{x}) \mapsto u(y) + \mathbf{b}^{\top} \mathbf{x}$ . One can also test whether u'(y) is indeed independent of  $\mathbf{x}$ .

LMN's regressions cannot, however, determine the form of the ethical NMUF  $(y, \mathbf{x}) \mapsto v(y, \mathbf{x})$ beyond some increasing transformation  $\phi(u(y) + \mathbf{b}^{\top}\mathbf{x})$ of the empirically determined stochastic utility function.

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#### 2 Precursors





#### 5 General SWB





### Welfare Weights and Inequality Aversion

Psychological facts concerning SWB measures cannot by themselves determine:

- welfare weights which tell us how to trade off some individuals' welfare gains against others' losses;
- inequality aversion concerning the elasticity of the marginal rate of substitution between different individuals' incomes.

These both require much stronger ethical value judgements, giving the probabilities P(s|z, t) (questionable) cardinal significance as welfare indicators.

This is implicitly what Layard, Mayraz and Nickell in particular do.



# Arrow's Questions

#### 1951 Is there a satisfactory voting procedure for aggregating individual values into a "rational" social objective?

2012 What has this talk to do with the Easterlin paradox?

Perhaps more income does increase SWB.

Perhaps, however, the "paradox" arises because individuals adapt to higher income levels, and expect continued growth.

The static theory discussed here says nothing about this.



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### Robert F. Kennedy

Address, University of Kansas, Lawrence, Kansas, March 18, 1968:

"Too much and too long, we seem to have surrendered community excellence and community values in the mere accumulation of material things. Our gross national product ... if we should judge America by that — counts air pollution and cigarette advertising, and ambulances to clear our highways of carnage. It counts special locks for our doors and the jails for those who break them. It counts the destruction of our redwoods and the loss of our natural wonder in chaotic sprawl. It counts napalm and the cost of a nuclear warhead, and armored cars for police who fight riots in our streets. It counts Whitman's rifle and Speck's knife, and the television programs which glorify violence in order to sell toys to our children."

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### Robert F. Kennedy, continued

"Yet the gross national product does not allow for the health of our children, the quality of their education, or the joy of their play. It does not include the beauty of our poetry or the strength of our marriages; the intelligence of our public debate or the integrity of our public officials. It measures neither our wit nor our courage; neither our wisdom nor our learning; neither our compassion nor our devotion to our country; it measures everything, in short, except that which makes life worthwhile. And it tells us everything about America except why we are proud that we are Americans."

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4th July 1776: "We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain inalienable Rights, that among these are Life, Liberty and the pursuit of Happiness."

We should provide people, not with happiness, but with the opportunity to **pursue** happiness.

Relationship with Amartya Sen's work on individual capabilities?

But not just aggregate happiness.

Much work to do.



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