## Structural

 Equation ModellingShort course in Applied Psychometrics

## This course

The course is funded by the ESRC RDI and hosted by The Psychometrics Centre

漛圈 UNIVERSITY OF
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The Psychometrics Centre
Researcher Development Initiative

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|  | Day 1 | Day 2 | Day 3 |  |
| :---: | :---: | :---: | :---: | :---: |
| 9:00- | Coffee on arrival | Lec-6 - Special issues in CFA <br> Correlated errors <br> Bi-factor modelling <br> Method factors <br> Multi-group CFA | Lec-9 - SEM <br> Incorporating latent traits into path models. | 9:00- |
| 9:20- | Introductions + Aims of course |  |  | 9:20- |
| 9:40- | Lec-1 <br> Mplus modelling framework |  |  | 9:40- |
| 10:00- |  |  |  | 10:00- |
| 10:20- |  |  |  | 10:20- |
| 10:40- |  |  |  | 10:40- |
| 11:00- | Coffee | Coffee | Coffee | 11:00- |
| 11:20- | Lec-2 - Regression models | Lec-7 - Path models 1 <br> The basics / figures / Identification/ model fit/ equivalent models | Examples 5 - SEM | 11:20- |
| 11:40- |  |  | EAS - SEM | 11:40- |
| 12:00- | Examples 1 <br> EAS - regression models |  | Wrapping up, further reading and | 12:00- |
| 12:20- |  |  | questions | 12:20- |
| 12:40- |  | Examples 3: SZ paper. | Lunch and depart | 12:40- |
| 13:00- | Lunch | Lunch |  | 13:00- |
| 13:20- |  |  |  | 13:20- |
| 13:40- |  |  |  | 13:40- |
| 14:00- | Lec-3-CFA with continuous variables | $\frac{\text { Lec-8 - Path models } 2}{\text { Model refinement }}$ |  | 14:00- |
| 14:20- |  |  |  | 14:20- |
| 14:40- |  |  |  | 14:40- |
| 15:00- | Lec-4 - EFA with continuous | Direct and indirect effects Binary mediators - logit/probit |  | 15:00- |
| 15:20- |  |  |  | 15:20- |
| 15:40- |  |  |  | 15:40- |
| 16:00- | Coffee | Coffee |  | 16:00- |
| 16:20- | $\frac{\text { Lec- } 5-\text { CFA and EFA with }}{\underline{\text { categorical variables }}}$ | Examples 4 |  | 16:20- |
| 16:40- |  |  |  | 16:40- |
| 17:00- | Examples 2 |  |  | 17:00- |
| 17:20- | EAS - CFA/EFA | Path model using EAS |  | 17:20- |
| 17:40- |  |  |  | 17:40- |

# Mplus model syntax 

A quick refresher

## Model statements: BY / WITH / ON



OX is correlated WITH Y
X with Y ;

$\bigcirc \mathrm{Y}$ (outcome) is regressed on X (predictor) Y on X ;

of (the factor) is measured BY Y1 Y2 Y3 F by Y1 Y2 Y3;

## Variable means

0 Stuff in a square bracket is a mean/intercept:
[wt_7 wt_9 wt_11];
o It's just the same to say:
[wt_7];
[wt_9];
[wt_11];

## Variances

o No bracket, then it's a variance / residual variance:
wt_7;
wt_9;
wt_11;
o Or
wt_7 wt_9 wt_11;

## Parameter equality constraints

o Three residual variances constrained to be equal:

$$
\begin{array}{ll}
\text { wt_7 } & \text { (1); } \\
\text { wt_9 } & \text { (1); } \\
\text { wt_11 } & \text { (1); }
\end{array}
$$

- Three intercept constrained to be equal:
[wt_7] (2);
[wt_9] (2);
[wt_11] (2);


## Parameter equality constraints

o Three residual variances constrained to be equal:

$$
\begin{array}{ll}
\text { wt_7 } & \text { (fixvar); } \\
\text { wt_9 } & \text { (fixvar); } \\
\text { wt_11 } & \text { (fixvar); }
\end{array}
$$

- Three intercept constrained to be equal:
[wt_7] (fixmean);
[wt_9] (fixmean);
[wt_11] (fixmean);


## Fixing parameters

- Constraining a covariance to be zero: X with Y@0;
o Constraining a mean to be zero:
[wt_7@0];
- Constraining a variance to be zero: i@0;


# Dataset for exercises 

EAS-ing you into SEM

## Where ALSPAC is based



## What is ALSPAC?

o Avon Longitudinal Study of Parents and Children AKA "Children of the Nineties"
o Cohort study of $\sim 14,000$ children and their parents, based in South-West England
o Eligibility criteria: Mothers had to be resident in Avon and ha an expected date of delivery between April $1^{\text {st }}$ 1991 and December 31 ${ }^{\text {st }} 1992$

O Population-Based Prospective Birth-Cohort

## What data does ALSPAC have?

oSelf completion questionnaires

- Mothers, Partners, Children, Teachers
oHands on assessments
- 10\% sample tested regularly since birth
$\circ$ Yearly clinics for all since age 7
o Data from external sources
o SATS from LEA, Child Health database
oBiological samples
- DNA / cell lines

How often is your child's behaviour like that given below:

|  | Never | Rarely | Some- <br> times | Often |
| :--- | :--- | :--- | :--- | :--- |

## Full set of ordinal EAS items

| act_t1_1 | Always on the go (+) | shy_t1_1 | Shy (-) |
| :---: | :---: | :---: | :---: |
| act_t1_2 | Moves about slowly (-) | shy_t1_2 | Makes friends (+) |
| act_t1_3 | Active on waking (+) | shy_t1_3 | Sociable (+) |
| act_t1_4 | Very energetic (+) | shy_t1_4 | Takes time warming to strangers (-) |
| act_t1_5 | Prefers quiet games (-) | shy_t1_5 | Friendly with strangers (+) |
| emo_t1_1 | Cries easily (-) | soc_t1_1 | Likes being with people (+) |
| emo_t1_2 | Emotional (-) | soc_t1_2 | Prefers playing with others (+) |
| emo_t1_3 | Often fusses and cries (-) | soc_t1_3 | Finds people stimulating (+) |
| emo_t1_4 | Gets upset easily (-) | soc_t1_4 | Something of a loner (-) |
| emo_t1_5 | Reacts intensely when upset (-) | soc_t1_5 | Isolated when alone (+) |

## Possible predictors

| Sex | Gender (1=Male, 2=Female) |
| :---: | :---: |
| mumage | Maternal age at delivery ( $1=<25,2=25-29,3=30-34,4=35+$ ) |
| tenure | Housing tenure ( $0=$ mortgaged, $1=$ private rented, $2=$ subsidized rented $)$ |
| crowding | Home overcrowding (> 1 person per room; 0=no, 1=yes) |
| parity | Parity ( $0=1^{\text {st }}$ born, $1=2^{\text {nd }}$ born, $2=3{ }^{\text {rd }}$ born + ) |
| mumed | Maternal educational attainment ( $0=$ A-level+, $1=$ O-level, $2=<$-level $)$ |
| income | Household income ( $0=$ bottom 20\%, 1 = middle 60\%, $2=$ top 20\%) |
| social | Social class ( $0=$ I/II, $1=$ III non-manual or lower) |
| mumalc | Regular maternal alcohol use in the early postnatal period (0=no, 1=yes) |
| mumsmk | Maternal cigarette use in the early postnatal period (0=none, 1=low, 2=high) |
| mdep_pn | Mother exceeding threshold for EPDS in early postnatal period (0=no, 1=yes) |

## Basic Input file (courtesy of "Stata2mplus")

```
Data:
    File is H:\Courses\SEM_2012\data\eas_1500.dta.dat ;
Variable:
    Names are id
        sex
        act_t1_1 act_t1_2 act_t1_3 act_t1_4 act_t1_5
        emo_t1_1 emo_t1_2 emo_t1_3 emo_t1_4 emo_t1_5
        <snip>
        shy_t3_1 shy_t3_2 shy_t3_3 shy_t3_4 shy_t3_5
        soc_t3_1 soc_t3_2 soc_t3_3 soc_t3_4 soc_t3_5
        mumage tenure crowding parity mumed income social mumalc mumsmk mdep_pn
        mfq10_01 mfq10_02 mfq10_03 mfq10_04 mfq10_05 mfq10_06
        mfq10_07 mfq10_08 mfq10_09 mfq10_10 mfq10_11 mfq10_12 mfq10_13
        mfq18_01 mfq18_02 mfq18_03 mfq18_04 mfq18_05 mfq18_06
        mfq18_07 mfq18_08 mfq18_09 mfq18_10 mfq18_11 mfq18_12 mfq18_13
        emotott1 emotott2 emotott3 acttott1 acttott2 acttott3
        shytott1 shytott2 shytott3 soctott1 soctott2 soctott3;
    Missing are all (9999) ;
Analysis:
    Type = basic ;
```


## Logit and Probit models

## Logit and Probit models

oBoth are latent variable models of sorts o Observed binary variable Y oAssumed underlying continuous variable $\mathrm{Y}^{*}$ oVariance of $Y^{*}$ is unknown

## OLS Regression model



Variance of Y is known
Residuals are assumed to be standard normal
Aim is to use covariates to explain the variance in $Y$ Residual variance will typically reduce

## Logit and Probit models



Variance of Y is unknown
Residuals assumed to be standard normal / logistic Residual variance is usually FIXED at one or $\pi^{2} / 3$ Otherwise the scaling is arbitrary

## Normal and Logistic distributions



Fig. 25. Logistic versus normal distribution with matched peaks, for $r=1, t^{*}=0$, $\dot{m}=1 / 4$.

TW. Patzek, G.D. Croft / Energy 35 (2010) 3109-3122

## Quick example

## Data:

File is "C:\Work\SEM Course\eas_1500.dta.dat" ;
Define:

$$
\begin{aligned}
\text { mfqsum18 } & =\text { mfq18_01 }+ \text { mfq18_02 }+ \text { mfq18_03 }+ \text { mfq18_04 + mfq18_05 } \\
& + \text { mfq18_06 }+ \text { mfq18_07 }+ \text { mfq18_08 }+ \text { mfq18_09 }+ \text { mfq18_10 } \\
& + \text { mfq18_11 }+ \text { mfq18_12 }+ \text { mfq18_13; }
\end{aligned}
$$

mfqcase = (mfqsum18>11);
emomean $=($ emotott1+emotott2+emotott3)/3;
Variable:
Names are id etc....
Usevariables = mfqcase emomean;
Categorical = mfqcase;

## ML Logit model

Analysis:
estimator = ML;

Model:
mfqcase on emomean;
Output:
cint;

## Logit model results



## WLSMV Probit model

Analysis:
estimator = WLSMV;

Model:
mfqcase on emomean;
Output:
cint;

## ML Probit model

Analysis:
estimator = ML;
link = probit;
Model:
mfqcase on emomean;
Output:
cint;

## Comparison of results

○ ML Probit

|  | Estimate | S.E. | Est./S.E. | P-Value |
| :--- | ---: | ---: | ---: | ---: |
| MFQCASE ON EMOMEAN | 0.532 | 0.124 | 4.279 | 0.000 |
| Thresholds - MFQCASE\$1 | 1.406 | 0.107 | 13.198 | 0.000 |

- WLSMV Probit

|  | Estimate | S.E. | Est./S.E. | P-Value |
| :--- | ---: | ---: | ---: | ---: |
| MFQCASE ON EMOMEAN | 0.531 | 0.124 | 4.272 | 0.000 |
| Thresholds - MFQCASE\$1 | 1.406 | 0.107 | 13.185 | 0.000 |

- ML Logit

MFQCASE ON EMOMEAN
Thresholds MFQCASE\$1

| Estimate | S.E. | Est./S.E. | P-Value |
| ---: | ---: | ---: | ---: |
| 0.954 | 0.222 | 4.300 | 0.000 |
| 2.403 | 0.195 | 12.293 | 0.000 |

## Logit and probit models

o Probit - ML and WLSMV give almost identical results

0 Logit and Probit params have different interpretation however models are very similar
o Not statistical criteria for choosing between probit/logit
o Down to preference and research discipline
o Fixed scales (important when we come on to mediation)

# Time for practical \#1 

Regression models in Mplus

