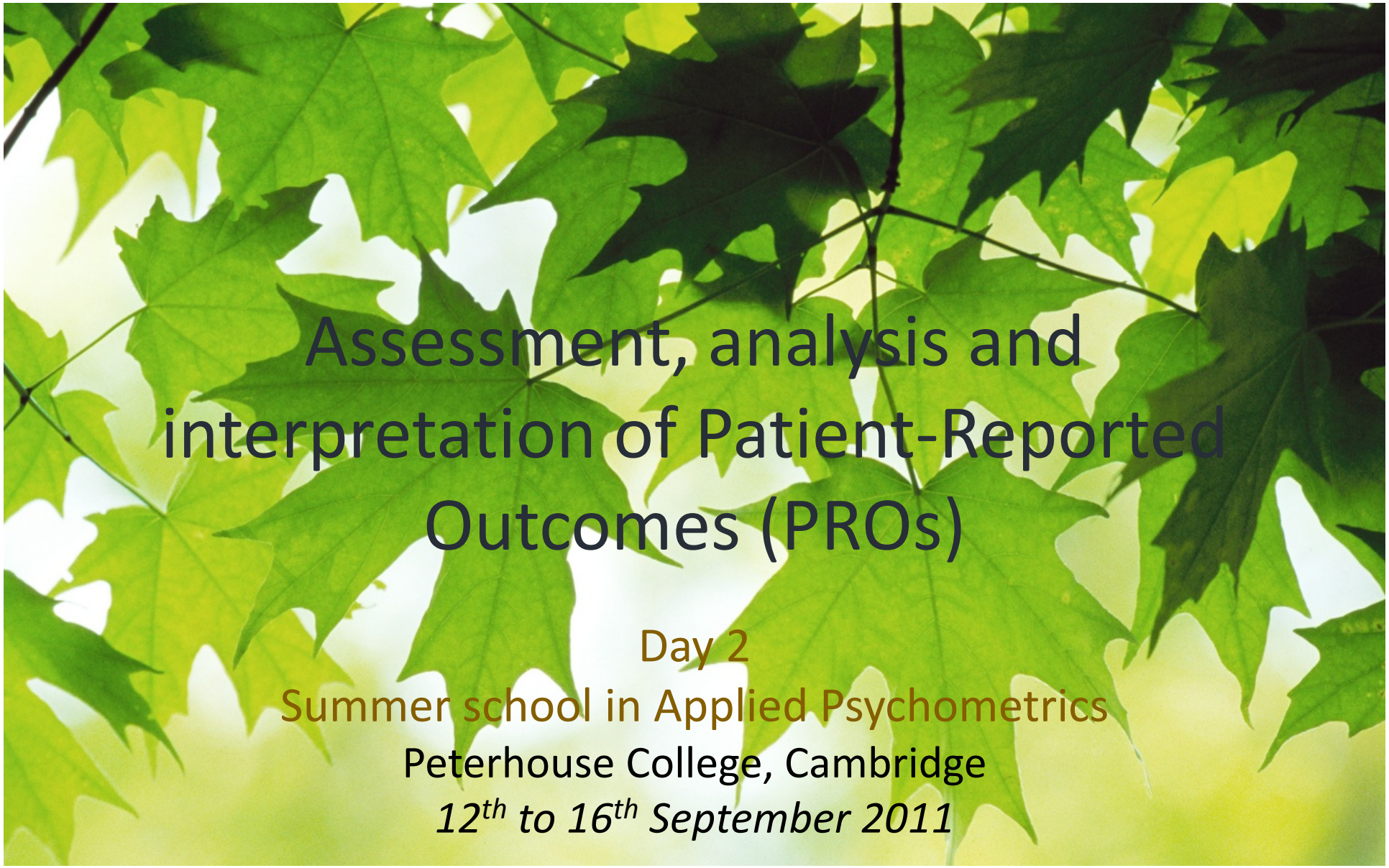

Assessment, analysis and interpretation of Patient-Reported Outcomes (PROs)

Day 2

Summer school in Applied Psychometrics

Peterhouse College, Cambridge

12th to 16th September 2011

UNIVERSITY OF CAMBRIDGE
The Psychometrics Centre

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5. STRUCTURE AND QUALITY OF SCALES

The construct validity question

- A central issue in PRO measurement is whether obtained scores represent the measured trait (e.g., severity of depression).
- The empirical question is whether the relationships among scale items can be explained by a single underlying trait (e.g., depression),
  - and are thus unidimensional,
  - or form sub-scales to operationalize the trait's multidimensional structure.
    - (Gibbons, Immekus and Bock, 2007)

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Test homogeneity

- Homogeneous test is the test whose items are all of the same kind – they measure something in common
- That "something" is an attribute we want to measure
- Quantity measured in common is a *common factor*, F
- Not all items are equally good indicators of F

Homogeneity and symptom clusters

- Homogeneity means that all items are parallel indicators of the latent attribute
- In QoL measures, some symptoms may form clusters related to a common cause
  - E.g. chemotherapy in cancer causes nausea and hair loss. These are highly correlated but might be unrelated to each other in terms of QoL
- Care is needed to identify such items and treat them appropriately from the validity perspective

Validity assessment

- Scale development
  - Expect reasonable correlations between items in the scale, and consequently between the item and its own scale
  - During development, correlation between item and its scale should be 0.3 or greater
  - At the final stages, more stringent criterion of 0.4 could be applied
  - Discriminant validity is supported if the item correlates higher with its scale than with other scales
- Construct validity of multiple scales
  - Convergent and discriminant validity

Correlation coefficients

- Relationships between items, and between items and the scale are usually described by correlations
- Sample size required for classical methods is at least 100, important to check for restriction of range
- Here are some correlation coefficients widely used in *classical* analysis of questionnaire data
  - Pearson's correlation coefficient assumes normally distributed variables
  - Spearman's rank correlation is more suitable for ordered categorical variables
  - Biserial coefficient is suitable for estimating correlation between continuous and binary variable (with underlying normal distribution assumed)

Common factor model

- The appropriate model is the common factor model of Spearman

$$Y_i = \mu_i + \lambda_i F + E_i$$

  - Constants μ allow each item to have unique difficulty
  - Factor loadings λ reflect the difference in response corresponding to difference in attribute – therefore it is a measure of item discrimination
  - Special case is true-score equivalent items

$$(\lambda_l = \lambda_k = \lambda)$$

Dimensionality and Factor analysis

- Unidimensionality is another name for homogeneity
- We can check if items are of homogeneous content – measure just one attribute in common – by checking if responses fit the single factor model
  - This can be done after model parameters are estimated
  - We can compute discrepancies between observed and expected covariances, and summarize them (for example) as ordinary mean of squared differences (ULS function)
  - We can examine overall fit, and any areas of misfit (violating covariances)
- Factor analysis is concerned with structure of correlations (between items or scales)

Reliability of a homogeneous test

- The total summated test score is

$$Y = \Sigma\mu_i + (\Sigma\lambda_i)F + \Sigma E_i$$

- Variance of test score can be partitioned

$$\sigma_Y^2 = (\Sigma\lambda_i)^2 + \Sigma\,\sigma_{Ei}^2$$

- Reliability is defined as
  - *squared correlation between the observed and the true score*
  - or, as proportion of variance due to true score
- Coefficient omega (Browne, 1975)

$$\omega = (\Sigma\lambda_i)^2 / \sigma_Y^2$$

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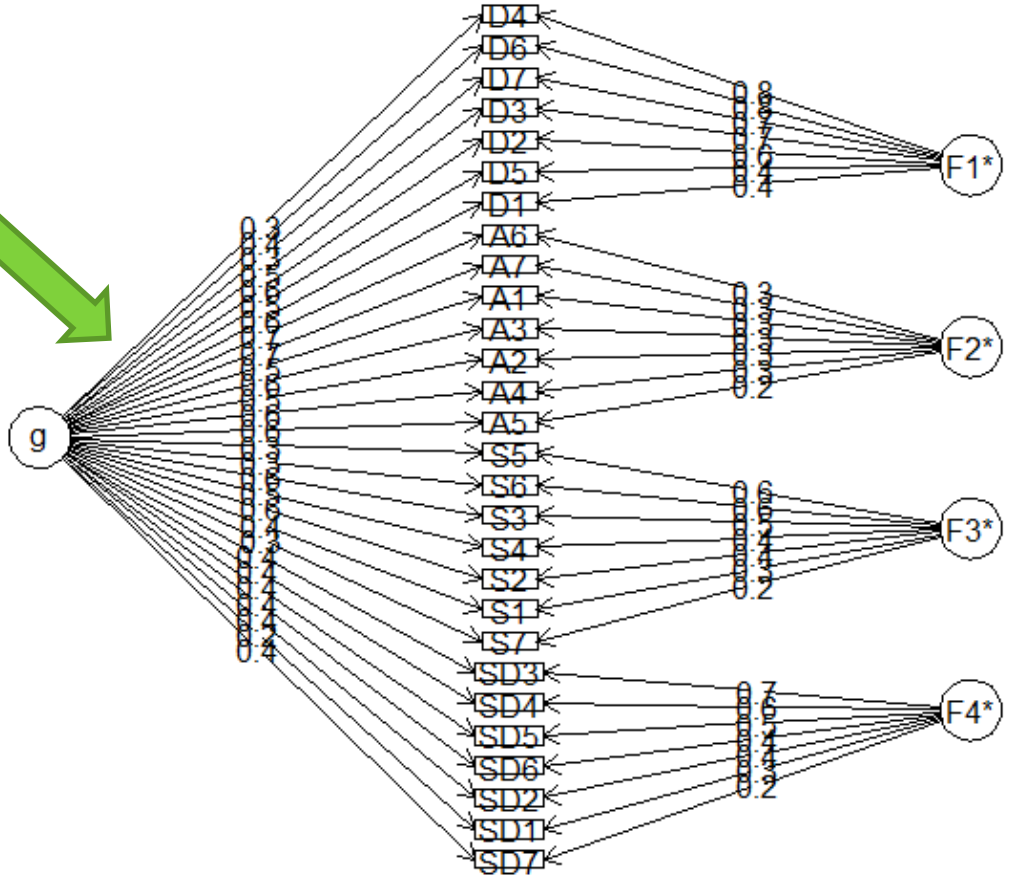
Coefficient *omega*

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_{gi}\right)^2}{\mathrm{var}(y)}$$

- Based on the common factor model
- Works whether one or more factors are present
- λ_{gi} are factor loadings on the general factor



Coefficient *alpha*

- For true-score equivalent items the expression for coefficient omega reduces to

$$\omega_\tau = m^2 \lambda^2 / \sigma_Y^2 = m^2 \sigma_T^2 / \sigma_Y^2$$

  - where σ_T^2 is covariance between any two items
- Sample-based estimates of population values

$$\hat{}\sigma_T^2 = (\Sigma\Sigma s_{ik} - \Sigma s_{ii})/[m(m-1)]$$

$$\hat{}\sigma_Y^2 = \Sigma\Sigma s_{ik}$$

- we obtain coefficient alpha

$$\omega_\tau = [m/(m-1)]*(1 - \Sigma s_{ii}/ \Sigma\Sigma s_{ik}) = \alpha$$

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Do not misuse alpha

- In a unidimensional test, adding items usually increases alpha
- Alpha has been used as measure of "internal consistency" (homogeneity)
- Alpha itself provides no evidence of homogeneity
  - Applying alpha itself requires a lot of assumptions
  - Such as that the single-factor model holds
  - It is an estimate of reliability for true-score equivalent items
  - Computing alpha is no substitute for dimensionality assessment
  - Dimensionality should be always assessed first

Applying alpha and omega

- It can be shown that alpha is a lower bound to omega

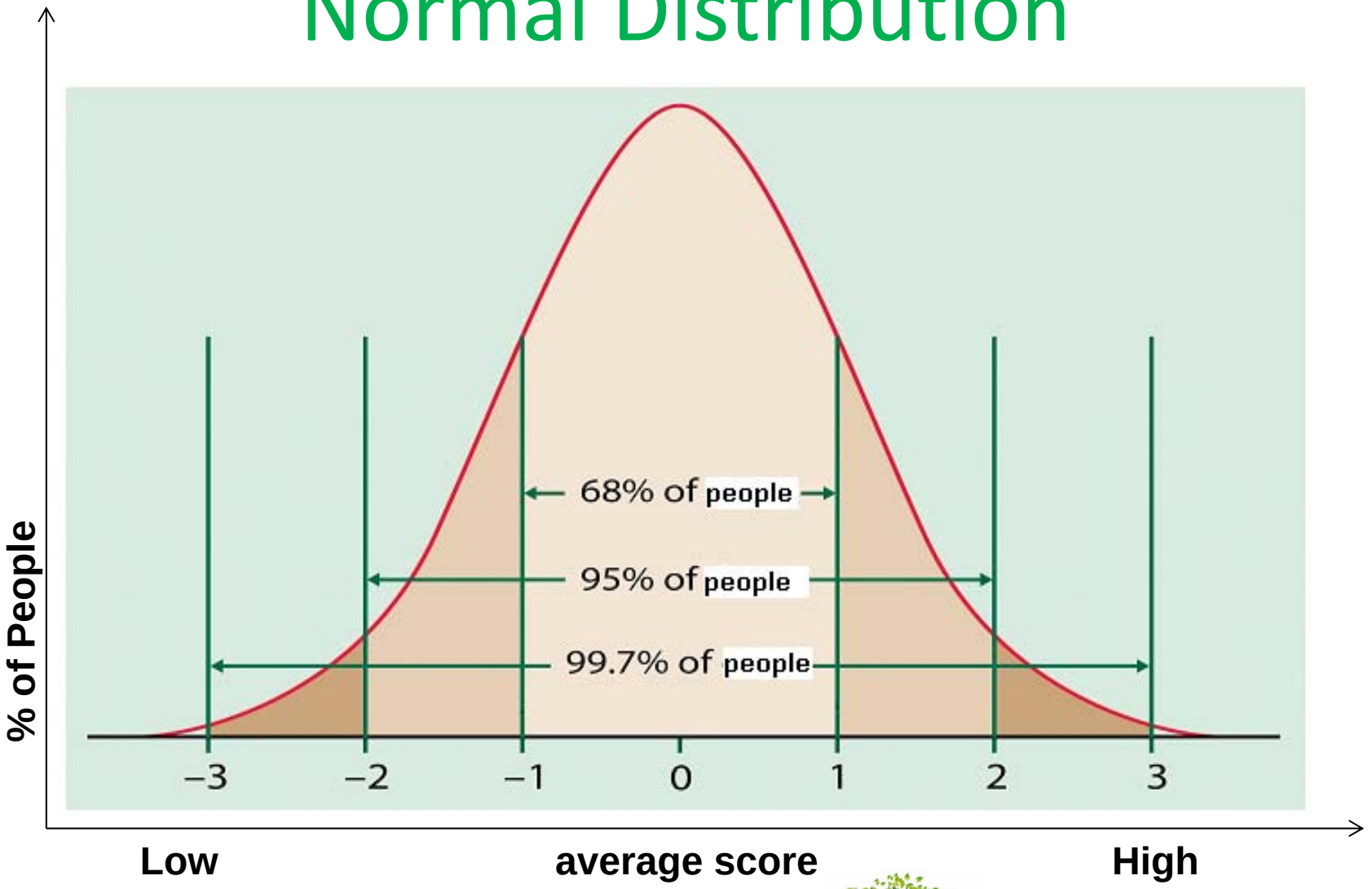
$$\omega_\tau \geq \alpha$$

- If unidimensionality holds in sum-scored test
  - alpha can be used as a crude tool to estimate the lower bound to reliability
  - If the factor model is fitted, omega is a better measure of reliability
- Guidelines to "sufficient" alpha or omega are meaningless without understanding what precision of measurement is required in a particular application
  - SE of measurement is more informative

Binary (and ordinal) items

- All the above calculations hold for the linear factor model
  - That is, relationships between expected item and test score are assumed linear
  - These relationships are not linear for binary items (and for items with small number of response categories)
- To overcome this problem, it is assumed that item scores are observed manifestations of latent response tendencies
  - Continuous and assumed normally distributed
  - Item responses relate to response tendencies through threshold process
- Correlations between items
  - Tetrachoric correlation is suitable for two binary variables (with underlying normal distribution assumed)
  - Polychoric correlation is suitable for two ordinal variables (with underlying normal distribution assumed)

Normal Distribution



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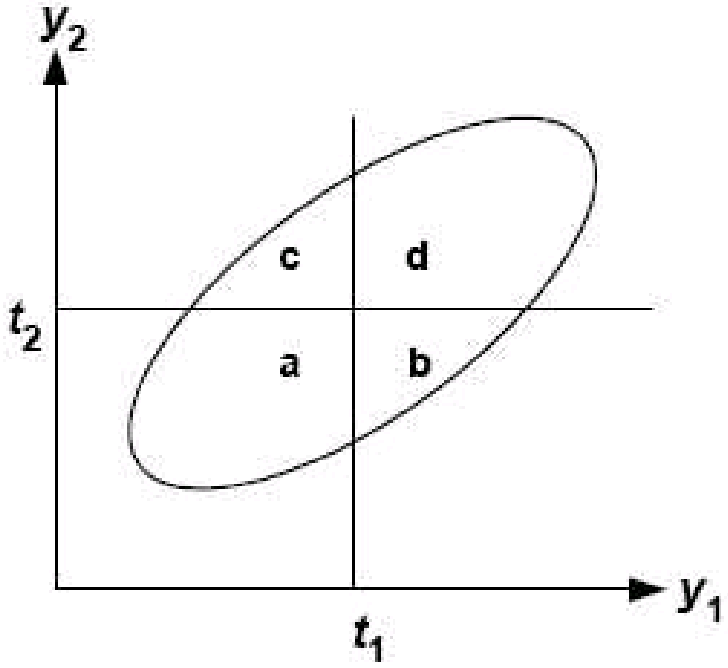
Tetrachoric correlation

	-	+	y1
-	a	b	a + b
+	c	d	c + d
y2	a + c	b + d	1



- Assumes that two normally distributed variables have been dichotomised using threshold process
- Appropriate assumption for item responses, where response tendency is unobserved, only its dichotomisation is observed

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Limitations of classical scale assessment

- Classical test theory assumes
  1. linear relationships between items and attributes
  - Relationships between binary (and ordinal) items and attributes are not linear
  - Linear model is an approximation only (still a useful summary, as summation of items tends to cancel the effects of non-linearity)
  2. independence of true and error variance
  - In general, the error variance of the total test score from a set of binary items cannot be independent of the test score