Topic A: Matrix Treatment of Simple Regression

Matrix Treatment of Regression

- Deals with the multiple regression case
- Estimators can be simply expressed in terms of matrices
- Most common way in which workings of panel models are described
- The population model is usually written:

Linear Model and the Least Squares Estimator in Matrix Terms

$$y = X\beta + \varepsilon$$

The least squares estimator is

$$\hat{\beta} = \left(X'X\right)^{-1} X'y$$

Simple Regression

The Data Matrix - X

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ . & . \\ 1 & x_n \end{bmatrix}$$

The Cross Product - X'X

$$\begin{bmatrix} 1 & 1 & 1 & . & 1 \\ x_1 & x_1 & x_1 & . & x_1 \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ . & . \\ 1 & x_n \end{bmatrix}$$

The Result

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix}$$

Least squares estimator
(Matrix Version)

$$\hat{\beta} = \left(X'X\right)^{-1} X'y$$

X'X

Inverse Matrix of X'X

Identity Matrix

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} \dfrac{\sum x_i^2}{n\sum x_i^2 - \left(\sum x_i\right)^2} & -\dfrac{\sum x_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \\ -\dfrac{\sum x_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} & \dfrac{n}{n\sum x_i^2 - \left(\sum x_i\right)^2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\sum x_i^2}{n\sum x_i^2 - \left(\sum x_i\right)^2} & -\dfrac{\sum x_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \\ -\dfrac{\sum x_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} & \dfrac{n}{n\sum x_i^2 - \left(\sum x_i\right)^2} \end{bmatrix} \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \\ \dfrac{-\sum x_i \sum y_i + n\sum x_i y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \end{bmatrix}$$

These are the standard formulae for the simple regression estimators!!

$$= \begin{bmatrix} \bar{y} - \hat{\beta}_1 \bar{x} \\ \dfrac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} \end{bmatrix} = \begin{bmatrix} \bar{y} - \hat{\beta}_1 \bar{x} \\ \dfrac{\sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum \left(x_i - \bar{x}\right)^2} \end{bmatrix}$$