A Boolean Approach to Qualitative Comparison

A summary by Richard Warnes

Charles Ragin (1987) THE COMPARATIVE METHOD. Chapter 6

The key features of Boolean Algebra:

1. USE OF BINARY DATA

"There are two conditions in Boolean Algebra: TRUE (or Present) and FALSE (or Absent). These two states are represented in base 2:

1 indicates presence;

0 indicates absence." Pg 86.

2. USE OF TRUTH TABLE TO REPRESENT DATA

"In order to use Boolean Algebra as a technique of qualitative comparison, it is necessary to reconstruct a raw data matrix as a truth table". Pg 87 eg:

X1	X2	X3	X4 (Conditions)	OUTCOME	NUMBER of INSTANCES
0	0	0	0	0	8
0	0	0	1	0	6
0	0	1	0	1	10
0	0	1	1	0	5
0	1	0	0	1	13

3. BOOLEAN ADDITION ('OR')

"In Boolean Algebra, if A + B = Z, and A = 1 and B = 1, then Z = 1. In other words 1 + 1 = 1. The basic idea in Boolean Addition is that if any of the additive terms is satisfied (Present), then the outcome is true (Occurs)." Pg 89.

Addition in Boolean Algebra is equivalent to the logical operator 'OR' ....Thus the statement A + B = Z becomes:-

If either A equals1 'OR' B equals 1, then Z equals 1.

4. BOOLEAN MULTIPLICATION ('AND')

"Boolean Multiplication differs substantially from normal multiplication. Boolean Multiplication is relevant because the typical social science application of Boolean Algebra concerns the process of simplifying expressions known as 'SUMS OF PRODUCTS'. A product is a specific combination of causal conditions eg.

F = Abc + aBc + abC + ABc + AbC + aBC + ABC

5. COMBINATORIAL LOGIC

"The absence of a cause has the same logical status as the presence of a cause in Boolean Analysis....Boolean Multiplication indicates that presence and absence conditions are combined...that they intersect." Pg 92.

6. BOOLEAN MINIMIZATION

"If two Boolean expressions differ in only one causal condition, yet produce the same outcome, then the causal condition that distinguishes the two expressions can be considered irrelevant and can be removed to create a simpler, combined expression." eg. Pg 93.

If Abc = F
and ABc = F

Then this allows the replacement of these two terms with the single, simpler expression:

Ac = F (ie. the value of B is irrelevant. Cause B may be either present or absent , F will still occur.)

7. IMPLICATION

"A Boolean Expression is said to imply another if the membership of the second term is a subset of the membership of the first". Pg 95.

eg. "A implies Abc because A embraces all the members of Abc (That is, Abc is a subset of A)...The membership of Abc is included in the membership of A. Thus A implies Abc."

"The concept of implication, while obvious, provides an important tool for minimizing primitive sums-of-products expressions." Pg 95.

8. PRIME IMPLICANTS and PRIME IMPLICANTS CHARTS

"The first step in the Boolean analysis of these data is to attempt to combine as many compatible rows of the truth table as possible (See 6. Boolean Minimization). Thus:

S = AbC + aBc + ABc + ABC (These are referred to as 'PRIMITIVE EXPRESSIONS')

ABC combines with AbC to produce AC.
ABC combines with ABc to produce AB.
ABc combines with aBc to produce Bc.

Thus S = AC + AB + Bc (These equations, reduced to their lowest level are referred to as 'PRIME IMPLICANTS')

"Product terms such as those in the preceding equation which are produced using this simple minimization rule - combine rows that differ on only one cause if they have the same output values - are called 'PRIME IMPLICANTS'. Usually each PRIME IMPLICANT covers (that is implies) several primitive expressions in the truth table. In the partially minimized equation given above, for example, prime implicant AC covers two primitive Boolean expressions listed in the truth table: ABC and AbC." Pg 96.

"In order to determine which prime implicants are logically essential, a minimization device known as a 'PRIME IMPLICANT CHART' is used. Minimization of the prime implicant chart is an optional, second phase of Boolean Minimization. The goal of this second phase of the minimization process is to 'cover' as many of the primitive Boolean expressions as possible with a logically minimal number of Prime Implicants." Pg 97. eg:

	ABC	AbC	ABc	aBc
AC	X	X		
AB	X		X	
Bc			X	X

"Simple inspection indicates that the smallest number of Prime Implicants needed to cover all of the original primitive expressions is two....Prime Implicants AC and Bc cover all four Boolean Primitive Expressions. Analysis of the Prime Implicant Chart, therefore, leads to the final reduced Boolean expression containing only the logically 'Essential Prime Implicants':
Thus

S = AC + Bc (These are the 'ESSENTIAL PRIME IMPLICANTS')

9. USE OF De MORGAN'S LAW

"It is often useful to assess the combinations of conditions associated with the absence of an outcome. Rather than start from the very beginning.....it is possible to apply De MORGAN's LAW to the solution already derived for positive outcomes to obtain the solution for negative outcomes." Pg 99.

Thus if S = AC + Bc to find the negative outcomes:- (Two Rules)

RULE 1: "Elements that are coded present in the reduced equation (Say A in the term AC) are recoded to absent, and elements that are coded absent (Say c in the term Bc) are recoded to present."

RULE 2: "Next, logical AND (x) is recoded to logical OR (+), and logical OR (+) is recoded to logical AND (x)."

Applying these two rules:

S = AC + Bc becomes:

S = (a + c) (b + C) which goes to:

S = ab + aC + bc

"De MORGAN's law thus provides a convenient shortcut for minimizing negative instances." Pg 99.

10. NECESSARY AND SUFFICIENT CAUSES

NECESSARY: "A cause is defined as necessary if it must be present for a certain outcome to occur".

SUFFICIENT: "A cause is defined as sufficient if by itself it can produce a certain outcome."

Thus if: S = AC + Bc (No cause is either necessary or sufficient)

S = AC + BC (C is necessary but not sufficient)

S = AC (Both A and C are necessary but not sufficient)

S = A + Bc (A is sufficient but not necessary)

S = B (B is both necessary and sufficient)

11. FACTORING BOOLEAN EXPRESSIONS

"Often it is useful to factor the results of Boolean analysis." Pg 100.

eg. if: $S = AB + AC + AD$

It can be factored to show that A is a NECESSARY CONDITION:

$S = A (B + C + D)$

or in a more complex equation

if: $S = abc + AbC + abd + E$

this can be shown as: $S = a (bc + bd + E) + A (bC + E)$

12. SUMMARY

"The approach has a strong inductive element (which mimics case orientated research) because it proceeds from the bottom up, simplifying complexity in a methodical, stepwise manner. It starts with a bias toward complexity - every logically possible combination of values is examined - and simplifies this complexity through experiment like contrasts.....Finally, it is highly compatible with the vocabulary of necessary and sufficient causation, a feature that enhances its value for assessing the limits of social scientific generalizations." Pg 101.