


QCA and Fuzzy Set Goodness-of-Fit Tests by Wendy Olsen

- Thanks to John McLoughlin for programming help in Python.
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- See also  <https://www.facebook.com/groups/mixednetwork/>
- Integrated Mixed Methods Network
- And www.compass.org
- And JISCMAIL QUAL-COMPARE (185 members)



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University of Bath



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1 Defining our terms and conceptual framework

- QCA=Qualitative Comparative Analysis
- QCA and fuzzy set comparative analysis is a set of systematic ways of studying causality.
- We make a simple data table of binary or ordinal variables.
- QCA helps discern necessary causality as well as sufficient causality.
- Any Sample Size, or whole population.
- QCA offers formal methods for analyzing contingency.

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A Conjunctural Logic Reflects The Nature Of The World

QCA, ... is conjunctural in its logic, examining the various ways in which specified factors interact and combine with one another to yield particular outcomes. " (Cress and Snow, 2000: 1079)

However... the world's conjunctures are subject to change at greater/lesser speeds ...

So our claims are **definite** with respect to the past/present
But **conjunctural and contingent** with regard to the future.

In these ways, the QCA analyst uses qualitative methods and assumes fluidity in the social world. "X affects Y" is also contingent on Z.

STRUCTURE DOXA HABITUS INSTITUTIONS EVENTS AGENCY
→ OUTCOMES
→ other changes in long run.

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How QCA Data Are Organised

- The Truth Table.
 - Crisp-Set Truth Table. All 0s and 1s.
 - Fuzzy sets involve measuring the degree of membership of a case in a set.
 - If any column is fuzzy, the whole thing is fuzzy.
 - One column can be used to count cases which are of the same overall configuration.
 - One column is set aside as the ‘outcome’.
- The NVIVO Approach.
 - The “casebook” in NVIVO.
 - The concept of multilevel cases.

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2 Empirical measure of Csuff (consistency) An Example. Cress and Snow ethnographic research in USA

- In 2000 the *American Journal of Sociology* published a QCA article which has become a standard reference work.
- The topic is the mobilisation of resources to help homeless people in USA.
- Their paper uses QCA very creatively by first of all noting (from their literature review) that four outcomes, not one, need to be taken into account. R1 R2 R3 R4 take up four columns of the data table.
- These outcomes are qualitatively compiled based on a series of ethnographic interactions with homelessness activists, homeless people, politicians and officials in 17 US cities. From the 17 cities of their research work, 8 were chosen for this paper’s QCA analysis. Among these 8 cities, 15 cases of Social Movement Organisations cover homelessness.
- The crisp-set QCA data table has 4 outcomes, 15 cases (rows), and about 8 causal factors. (12 columns in total)

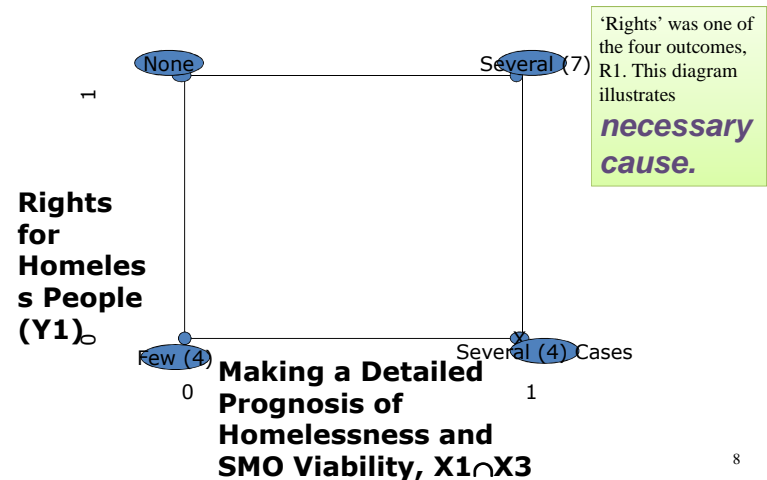
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Appendix: A Fuzzy Set Interim Truth Table (Olsen, 2009)

Y	X1	X2	X3	X4	X5	X6	Number Of Cases	Configuration
Fuzzy	Fuzzy	Fuzzy	Crisp	Crisp	Fuzzy	Crisp		
0	0	0	0	1	0	1	0	1
0	0	0	0	1	1	0	0	4
0	0	1	0	0	0	0	1	1
0	0	1	1	0	0	0	0	2
0	1	0	0	0	0	1	1	3
0	1	1	0	0	0	0	0	1
0	1	1	0	1	0	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	1	1	1
1	1	0	0	0	1	1	1	4
1	1	0	1	0	0	0	1	1
1	1	0	1	0	1	1	1	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	0	2	15
1	1	1	0	0	1	1	4	16
1	1	1	0	1	0	0	1	17

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Snow and Cress’s Findings Used Crisp Sets



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Snow and Cress's Findings

- There was no single pathway for a single outcome
- There was no universal causal pathway for the whole set of positive outcomes.
- Each **pathway** deserved, and got, ethnographic, observational (shadowing, buddying) treatment.

In this paper we offer software to measure the impact of X1 X2 X3 X4 X5 X6 on either Y1 Y2 Y3 or Y4.

JUST PUT YOUR DATA IN AND YOU GET GRAPHS AND CONSISTENCY VALUES OUT.

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A WARNING ABOUT COMPLETENESS OF CAUSAL MODELS

- (A) Necessary causes (B) Sufficient pathways
- You could practically remove the 'necessary causes' (call this X7 and X8) from the test for 'sufficient causes'.
- That's because the necessary causal factor is practically present in every case. So it does not affect the measurement or testing of X being sufficient for Y.

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3 Empirical measure of Goodness-of-fit (F)

Based on Eliason S. & Stryker R. 2009. Sociological Methods & Research 38:102-146.

Eliason & Stryker 2009 offered a test of fit to a hypothesis, e.g. that X is sufficient for Y.

Do the case-study research first,
Then crisp- or fuzzy-set QCA analysis,
Then notice which are the causal pathways
(A) Necessary causes (B) Sufficient pathways
Thirdly statistical testing.

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Appendix: A Fuzzy Set Interim Truth Table (Olsen, 2009)

Y	X1	X2	X3	X4	X5	X6	Number Of Cases	Configuration
Fuzzy	Fuzzy	Fuzzy	Crisp	Crisp	Fuzzy	Crisp		
0	0	0	0	1	0	1	0	1
0	0	0	0	1	1	0	0	4
0	0	1	0	0	0	0	1	1
0	0	1	1	0	0	0	0	2
0	1	0	0	0	0	1	1	3
0	1	1	0	0	0	0	0	1
0	1	1	0	1	0	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	1	1
1	1	0	0	0	1	1	1	4
1	1	0	1	0	0	0	0	1
1	1	0	1	0	1	1	1	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	0	1	0	2
1	1	1	1	0	0	1	1	4
1	1	1	1	0	1	0	0	1

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3.1 Empirical measure of Goodness-of-fit (F)

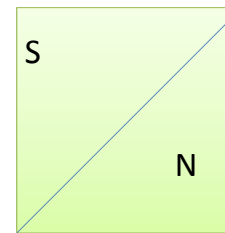
A Basic measure, C_{suff}

1. Is there a random sample? If you, consider statistical methods of testing. *Sociological Methodology 2015 debated this question.*
2. Follow Rihoux and Ragin's protocol.
2a) find what's Necessary. 2b) then Sufficient. 2c) then Converse.
3. For tests of sufficiency, you are now looking at joint membership in sets, known as $X1 \cap X2 \cap X3 = X$ etc.
 - A. The sufficiency triangle is the upper left area.
 - B. $\text{MIN}(X1, X2, X3)$ is the same as $X1 \cap X2 \cap X3$.
 - C. Eliason and Stryker advise to recalibrate into normal distributions.
4. You are now looking at individual X's first, and then at configurations that embed these. Thus the **effects are found to occur in combinations, known as configurations.**

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Visualising the Csuff Criterion

- The Consistency measure depends on the slopes of the lines that reach each point in the lower triangle. So it uses the vertical distances to the Diagonal in a crucial way.



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Rihoux and Ragin offer this measure of goodness of fit:

$$C_{suff} = \text{Consistency} = \text{Sum}(X \cap Y) / \text{Sum}(X) \quad \text{Eq. 1}$$

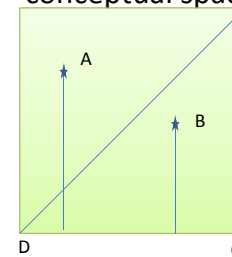
You sum over the cases. If $Y < X$, then the numerator, $\sum \text{Min}(X, Y)$, is less than the denominator.

For patterns with many cases lying in the Sufficiency Triangle, C_{suff} is =1 or close to 1. The cutoff point recommended by Ragin is 0.8, or 0.75.

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A Fuzzy Set Measure of Fit, C_{suff}

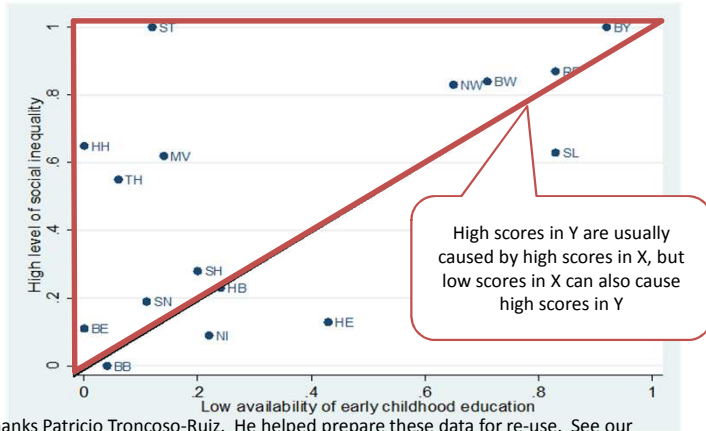
- Point A adds 1 unit to the numerator and denominator of C_{suff} . At Point B, the Y value is less than X. So it only adds to the denominator.
- Notice the fuzzy set space $\{0,0\}$ to $\{1,1\}$. This conceptual space is not Euclidean.



- A point represents a case.
- From B to the Diagonal is a non-zero distance. $C_{suff} < 1$ because of B.
- Suppose C is a case at (1,0)
- From C to the Diagonal is 1 unit! Huge.
- Suppose D is a case at (0,0)
- From D to the Diagonal is distance 0.
- D counts as 'in' the triangle S.

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Sufficiency of low availability of early childhood education for high level of social inequality



Thanks Patricio Troncoso-Ruiz. He helped prepare these data for re-use. See our Github area, to do this estimate yourself!

A More Advanced Measure of Fit, D_{suff}

- Will you consider that the fuzzy-set measurements could have measurement error?
 - If so: frequentist discourse

Ragin suggested softening the C_{suff} criterion for this very reason. See Ragin (2000).

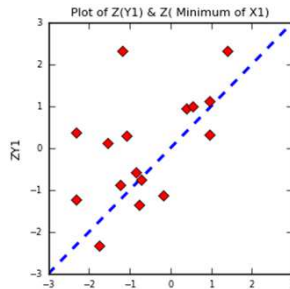
- If not: qualitative and realist discourse.
- A realist however *can* also use the frequentist discourse. Measurement error can be modelled.
 - If sampling cases: then in a probabilistic way, as descriptive of the data. We can reveal patterns in the population.
 - If not sampling cases: then in a hypothetical way.

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A German-Regions Education Illustration

Using our Python Freeware Program

Freitag, M., & Schlicht, R. (2009). Educational Federalism in Germany

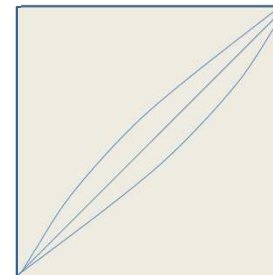


caseid	Late Education	Social Inequality
SH	0.2	0.28
HH	0	0.65
NI	0.22	0.09
HB	0.24	0.23
NW	0.65	0.83
HE	0.43	0.13
RP	0.83	0.87
BW	0.71	0.84
BY	0.92	1
SL	0.83	0.63
BE	0	0.11
BB	0.04	0
MV	0.14	0.62
SN	0.11	0.19
ST	0.12	1
TH	0.06	0.55

The pattern suggests that X is sufficient for Y with 5 exceptions. The consistency C_{suff} is .876. This meet's Ragin and Rihoux's criterion.

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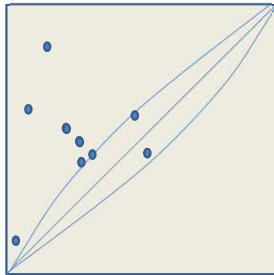
Stryker and Eliason allow for 0.1 average deviation at the middle of the fuzzy set space



The basis for this is that there could be error in any point in the graph, ie any case could have measurement error. They mention this could arise from inter-rater disagreement or from not having a firm basis for the fuzzy set membership score.

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Another illustration of Eliason & Stryker's concept of measurement error



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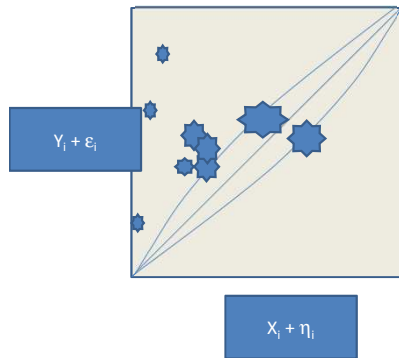
Next Activity (You may emulate this in any programming environment):

- Create gaussian variables for the configuration $X = X1 \cap X2 \cap X3$ and for Y .
- Using STATA, Python, R, SPSS, or Excel, calculate the D value: is the case in the sufficiency triangle, or not?
- -- if so, then $D=1$. If not, then $D=0$.
- --Multiple $(1-D)$ by the distance up to the "diagonal". $(1-0)$ is used to retain Distances.
- --The 'diagonal' in Fuzzy Set space is being moved to a new diagonal line in $Zx-Zy$ space.

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How to Set up Random Errors for Bootstrap Programme to get a credible interval around C_{suff} and D_{suff} See also Appendix Code.

FAR LEFT: Avg. Error=0. MIDDLE: Avg Error= $E(\eta_i) = 0.1$. FAR RIGHT: Avg. Error=0.



TOP: Avg. Error=0.

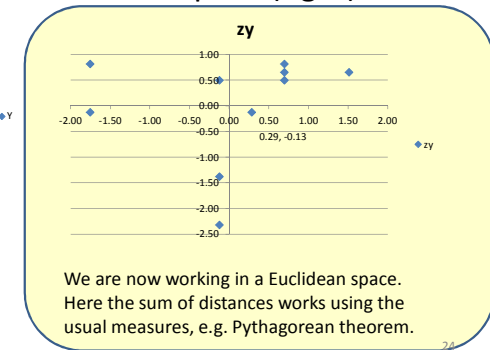
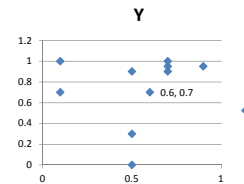
MIDDLE: Avg Error= $E(\epsilon_i) = 1$

BOTTOM: Avg. Error=0.

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A transformation

- Here some data is shown in the fuzzy set space (left) and the Z score space (right)



But we also added the Damper and the Measurement Error, and used InvCumNormal

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Eliason & Stryker Tricks

- Trick A: they convert the fuzzy set membership scores into normal distribution scores (Z-scores). To do this manually, you could subtract the mean and divide by the standard deviation.
- In a programme we use the inverse cumulative normal distribution to read off from Z score range the Z value that corresponds to this fuzzy set membership score. The X axis is read as a cumulative probability. Those cases with $X < 0.5$ get a Z value < 0 , and those on the right get a larger Z value.
- Trick B: they measure the distance from a case (Z_x, Z_y) to the diagonal line where $x=y$, and they note that $(y-x)^2$ gives this distance.
- Sum up these distances to get a measure of how far the cases disconform to the Suff hypothesis. The sum is called D_{suff} .

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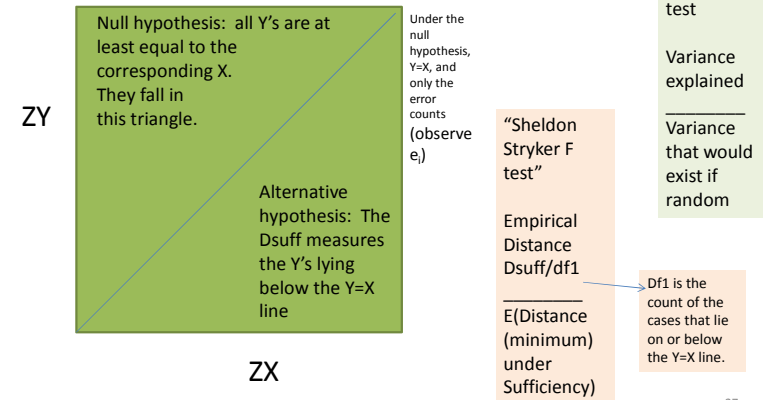
3.2 Empirical estimate of distance: Stryker's measure: $(1-D) * (z_y - z_x)^2$

- D is 1 if the case lies in the upper lefthand triangle.
 - D is 0 otherwise.
 - In PYTHON language:
`if (ylist[XL] > xlist[XL]): d = 1 else: d = 0`
 - Sum up the D_{suff} measure for all the cases in the group below the diagonal.
(If $D=1$ we multiply the distance by $1-D$ so that it is cancelled out.)
 - For example, if $N=30$ and 20 are above the diagonal, we are adding up 10 items to give the D_{suff} measure. D_{suff} is zero where $D=1$.
- (NOTE: Also, if $X=0$ for certain cases in a configuration, then cases should add nothing! !!) (By implication, if X is 0 for all cases, then that configuration is not causal on Y.)*

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Exploring the F Test for Sufficiency

- When D_{suff} is large, the evidence for a large F causes us to reject the null hypothesis of sufficiency of X for Y.



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Exploring Sufficiency Testing

- If the mean of Y and the mean of X give a point low down in the diagram, we tend to get a low Consistency level, depending on the skewness of the two variates.
- If the mean of Y and mean of X give a point high up in the diagram, the C_{suff} tends toward being large, and the D_{suff} tends toward being small.
- When D_{suff} is small, there's no need to reject the null hypothesis of X is sufficient for Y.
- A “ C_{suff} large” can be tested using the idea that the credible interval must not include 0.8.
- B “ D_{suff} small” can be tested using the F test claim that F is greater than the F cutoff. [OR that the c.i. for D_{suff} is small.
 - C We do not have a cutoff criterion for D_{suff} . Further research may suggest such a criterion value. The issue of measurement error must be taken into account, as well as the spread of X along the X axis.]

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Here is the formula and a description of the denominator

of the F test in Eliason and Stryker (2009)

$$F = \frac{SSD/DF1}{EMSD} = \frac{D_{suff}/DF1}{\text{Minimum Expected Distance if } H_0 \text{ is true}} \quad \text{Eq. 2}$$

- At the top is the distance for all the points, summed up, and standardised by DF1 (the N in the lower triangle).
- At the bottom is the distance **if the sufficiency of X for Y were found in the data** (without measurement error, this disappears as 0).
- At the bottom, it is not a unique distance, because many patterns are consistent with this.

The 'minimum distance under the null H'

$$\min(D_{null}) = \min(E\{D_{suff} | \text{causal sufficiency is true}\} / N$$

Eq. 3

(Eliason & Stryker, 2009, 115)

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Reminder: what sufficiency means.

- If X is sufficient for Y,
- Then **whenever X is non-zero, Y will be =X or greater.**
- Thus if X is 0, it is an irrelevant case for consistency in this sense.

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Eliason and Stryker say to consider measurement error.

- If the ZY and ZX are considered to be stochastic, then they may have both sampling error and measurement error. The idea of error here is that the sample may not give a perfect idea of the population. Then the true relationship cannot be known perfectly.
- Probability theory helps us know something about the pattern, and provides a 'confidence level'.
- P values are 100% - the conf. level
- E.g. 5% if the conf. level is 95% over repeat samples. Hence the rule $P < 0.05$ to 'reject'.

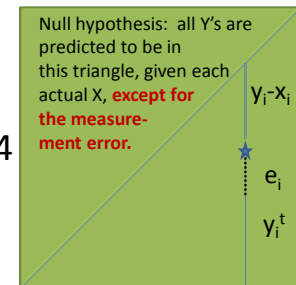
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What is the total distance in the numerator of the F?

- It's the sum of the individual distances from the point to the diagonal line, each squared before they're added up.

- The formula uses D_{suff}

$$\sum (1-D)(ZY - ZX)^2 \quad \text{Eq. 4}$$



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F statistic

A ratio of two r.v.s follows an F distribution if both r.v.s follow chi-squared distribution.

We see this in ANOVA and in the F test of Regression: If F is large, P is near 0 and we reject the null hypothesis, because the numerator exceeds the denominator more than it would by chance.

For our F statistic, the H_0 is: X is sufficient for Y.

Rejecting H_0 means we have X is NOT sufficient for Y. "Accepting" H_0 means we have not falsified H_0 .

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Interpretation of the denominator

- It is an innocuous feature, based on a null assumption.
- If F is large, there's a lack of support for the null hypothesis.
- If F is small, there's no way to reject the SUFF hypothesis. We want F small! 😊
- If F = 0 and X is always zero, you can't test causality of X.
- Watch out for remainders.

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This particular F Statistic

- When we take Z_x , this now becomes a point in space, so it does add something. The algebraic rules shift from Boolean to Euclidean.

F = msd/emsd on df1, df2 degrees of freedom. Eq. 2

= mean of the sum of Distance from Sufficiency / Expected Mean under Null Hypothesis

= $(\sum D_{suff} / df1) / E(e_e)$

WHERE: $msd = D_{suff} / df1$ Eq. 2a

And $emsd = nullsd$ Eq. 2b

D_{suff} = the sum of all $(1 - d) * (z_y - z_x)^2$ Eq. 4

$E(e_e) = nullsd = df2 * error_value^2$ Eq. 5

- The numerator arises as a measure of the observed distances from the hypothesized sufficiency relationship (which is independent of the denominator).
- The denominator is a measure of the expected value of the error in the model. The expectation of the squared errors.
- This error must be independent of X and Y. It is a piecewise linear function. Actually from a scalar 'Error_value' we want to generate the errors for each X but we have not allowed this correlation of X and error in this model. We follow Huang, R. <https://r-forge.r-project.org/scm/viewvc.php/pkg/QCA3/R/fsgof.R?view=markup&root=asrr> with error_value=0.05

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Illustrations

Config	Y	<i>C_{suff}</i>	<i>D_{suff}</i>	F	PVAL	Df1	Num
X1Y3	3	<u>1</u>	<u>0</u>	0	0	0	15
X2Y3	3	0.622	64.943	144.317	0	<u>3</u>	<u>15</u>

Note how Df1 gives a signal about coverage.

In such a case, configuration X1X3 is sufficient for Y. But there's no F test. All 15 cases support H_0 . There is no falsification.

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4 Empirical findings Real data illustrations

- Aims of this section:
- Show the graphs that our program makes.
- See <https://github.com/WendyOlsen/fsqof>
- Show that the Dsuff matches the Csuff in measuring the degree of deviation of the pattern from what would be expected if X were sufficient for Y.
- Show how an F test is interpreted for different sample sizes.
- Show how the degree of measurement error affects the test of goodness of fit.

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Indian village people's resistance to the landlord-employer's dictates

- Y4 is the key outcome reported on in Chapter by Olsen (2009) in Byrne & Ragin, eds.

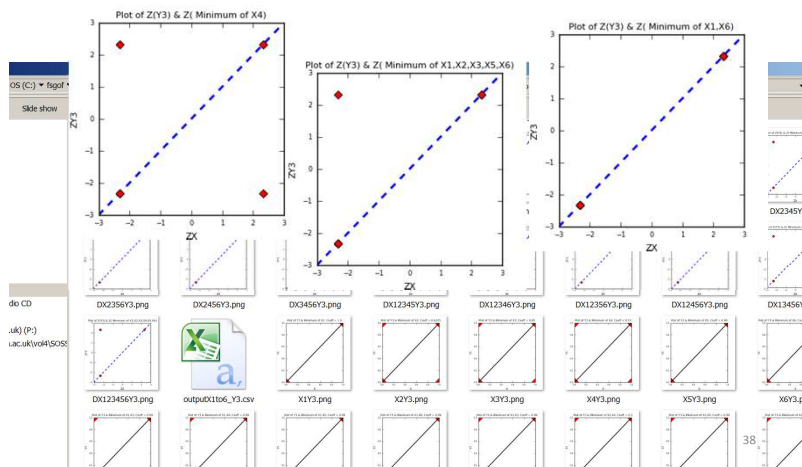
Handbook. Data sample:

hhid	worker	farmerl	assets	education	tenancy	wetaccess	havecows	conform	innovatn	resistfz
1	0	0	0.87	0.17	1	1	1	1	0	0
2	0	0	0.5	0.5	1	0	1	0	3	0.87
3	0	0	0.5	1	0	1	1	3	1	1
4	0	0	0.67	0.33	0	0	0	1	0	0.87
5	0	0	0.33	0.17	1	0.87	0	3	1	0
6	0	0	1	0.67	1	1	1	2	0	0

- Results (Sorted by Significance = Low)

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Cress & Snow (2000) Homeless Organisations Data

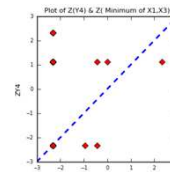


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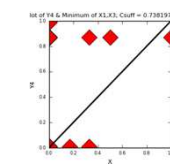
Do Boolean Algebra? (reduce) Only if the H_0 is **not rejected**.

(or use fsQCA freeware, see <http://www.u.arizona.edu/~cragin/fsQCA/software.shtml>)

Df1 is the number of exceptions.



Config	Y	Csuff	Dsuff	F	PVAL	Df1	Num
X13Y1	1	0.927	1.883	4.828	0.03	1	39
X45Y1	1	0.963	1.883	4.828	0.03	1	39
X134Y1	1	0.88	1.883	4.828	0.03	1	39
X345Y1	1	0.963	1.883	4.828	0.03	1	39
X456Y1	1	0.955	1.883	4.828	0.03	1	39
X3456Y1	1	0.955	1.883	4.828	0.03	1	39
X1Y1	1	0.4	129.885	55.507	0	6	39
X2Y1	1	0.623	64.943	55.507	0	3	39
X3Y1	1	0.769	72.701	23.302	0	8	39
X4Y1	1	0.581	119.836	20.485	0	15	39
X5Y1	1	0.925	21.648	55.507	0	1	39
X6Y1	1	0.798	76.864	49.272	0	4	39
X12Y1	1	0	0	0	0	0	39



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Boolean algebra rules

- If AB and Ab are associated with Y, then
- A(B or b) are associated with Y, so
- $A \rightarrow Y$ is justified as a simplification. (? Check your remainders, and your N and df1!). **Boolean reduction.**
- If AB and AC are associated with Y, then
- A(B or C) is a similar way to express this association. So A(B or C) can be tested for its overall sufficiency for Y. **Commutative, symmetrical? NO... if you again test using Not-Y your results may surprise you.**

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You may adjust the parameters.

- Error_value (Eliason and Stryker used 0.1)
 - Damping factor (default .01)
 - Labels on the output
 - Which Y you are studying: 1, 2, 3 or 4
- Send comments to wendy.olsen@manchester.ac.uk

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Config	Y	Csuff	Dsuff	F	PVAL	Df1	Num
X13Y1	1	0.927	1.883	4.828	0.03	1	39
X45Y1	1	0.963	1.883	4.828	0.03	1	39
X134Y1	1	0.88	1.883	4.828	0.03	1	39
X345Y1	1	0.963	1.883	4.828	0.03	1	39
X456Y1	1	0.955	1.883	4.828	0.03	1	39
X3456Y1	1	0.955	1.883	4.828	0.03	1	39
X1Y1	1	0.4	129.88%	55.507	0	6	39

- To illustrate 'reduction':
- $X1X3 + X4X5 + X1X3X4 + X3X4X5 + X4X5X6 + X3X4X5X6 \rightarrow Y$ implies:
- $X1X3 + X4*(X5 \text{ or } X1X3 \text{ or } X3X5 \text{ or } X5X6 \text{ or } X3X5X6)$.
- So in summary there are two pathways here.
- The Csuff suggested each of these is sufficient (Olsen, 2009).
- But the F test is falsifying this finding.
- Note also the role of a factor like X6. It, for example, is not necessary overall.
- But **X6 is an INUS condition if you use the C_{suff} criterion.**

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Conclusions

- If not a random sample, but purposive sampling, then it's unlikely that you should use a statistical test in an inferential framework. Use Ragin's Consistency measure.
- If it's a random sample, use both measures – Ragin's Consistency and the F test that Eliason and Stryker (2003, 2009) developed.
- If it's a whole population, you may use both, again, because there won't be a bias. You are allowing for measurement error or inter-rater disagreement. This is Eliason & Stryker's argument.

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Appendix 1A: A Fuzzy Set Interim Truth Table (Olsen, 2009)

Y	X1	X2	X3	X4	X5	X6	Number Of Cases	Configuration
Fuzzy	Fuzzy	Fuzzy	Crisp	Crisp	Fuzzy	Crisp		
0	0	0	0	1	0	1	0	1
0	0	0	0	1	1	0	0	4
0	0	1	0	0	0	0	1	1
0	0	1	1	0	0	0	0	2
0	1	0	0	0	0	1	1	3
0	1	1	0	0	0	0	0	1
0	1	1	0	1	1	0	1	7
0	1	1	1	1	1	1	1	8
1	0	0	0	0	0	0	1	9
1	1	0	0	0	1	1	1	4
1	1	0	1	0	0	0	0	1
1	1	0	1	0	1	1	1	12
1	1	0	1	1	0	1	1	13
1	1	0	1	1	1	1	1	14
1	1	1	0	0	0	1	0	2
1	1	1	0	0	0	1	1	4
1	1	1	1	0	1	0	0	1
1	1	1	1	0	1	0	0	17

Appendix 2: Ragin gave a Z score with a p value

- (Fuzzy Set Social Science, 2000)
- The p value is the risk of being wrong in rejecting a null hypothesis – here, the null is that the X is not sufficient for the Y.
- Each case has a p value.
- Each group of cases has a p value.
- Few scholars have emulated his Z test.
- Stryker and Eliason (2009) comment on a weakness of this test.

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Appendix 1B: A Fuzzy Set Raw Truth Table (Olsen, 2009) (White=X1-X6) (Purple=Y1-Y4)

work farm asset educ tenanweta haveccofo innov resist	hhid er	erll s	ation cy	ccess ows	rmn aten fz					
1	0	0	0.87	0.17	1	1	1	1	0	0
2	0	0	0.5	0.5	1	0	1	0	3	0.87
3	0	0	0.5	1	0	1	1	3	1	1
4	0	0	0.67	0.33	0	0	0	1	0	0.87
5	0	0	0.33	0.17	1	0.87	0	3	1	0
6	0	0	1	0.67	1	1	1	2	0	0
7	0	0	0.5	0.87	0	0	1	2	1	1
8	0	0	0.87	0.67	0	0.87	1	0	1	0
9	0	1	0.87	1	0	1	0	0	0	0.87
10	0	0	1	1	0	1	1	0	1	0.87
11	0	0	0.87	0.17	1	1	1	2	0	0.87
12	0	1	1	0.17	1	1	1	0	1	0.87
13	0	1	1	0.33	1	1	1	0	1	0
14	0	0	0.17	0	1	0	1	1	0	0.87
15	0	0	0.87	0.67	0	0	0	0	0	0
16	1	0	0.33	0.87	0	0	1	1	2	0
17	0	1	0.87	1	0	1	0	0	0	0.87
18	0	0	0.87	0.33	1	0	1	2	1	1
19	1	0	0	0.33	0	0	0	2	1	0
20	0	0	1	0.33	1	0.87	1	0	1	1
21	0	0	0.5	0	1	0	1	0	2	0.87
22	0	0	0.87	0.87	0	0	0	0	2	0.87
23	1	0	0	0.17	0	0	0	0	0	0
24	1	0	0	0.17	0	0	0	2	0	0.87
25	0	1	0.87	0.87	0	1	0	0	1	0
26	0	1	1	0.87	0	1	1	0	0	0.87
27	1	0	0.33	0.5	0	0	1	1	0	0.87
28	1	0	1	0.33	1	1	1	4	0	0.87
29	0	1	1	0.87	1	1	1	1	0	0.87
30	0	0	0.87	0.17	1	1	1	0	0	0.87

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Appendix 3: Snippet from Eliason and Stryker 2009

indicator (dummy) variable coded 1 when $y_i > x_i$ and 0 when $y_i \leq x_i$. For an xy biplot with $N(x_i, y_i)$ pairs, the accumulated squared Euclidean distance of the normalized fuzzy-set membership scores from that expected under each argument may now be defined.¹⁴

Squared distance from a null association:

$$D_{null} = \sum_{i=1}^N (z_{y(i)} - E\{Z_{y(i)} | \text{null XY association}\})^2$$
 Squared distance from causal necessity: $D_{nec} = \sum_{i=1}^N d_i (z_{y(i)} - z_{x(i)})^2$,
 Squared distance from causal sufficiency: $D_{suf} = \sum_{i=1}^N (1 - d_i) (z_{y(i)} - z_{x(i)})^2$,
 Squared distance from causal necessity and sufficiency:

$$D_{(nec \& suf)} = \sum_{i=1}^N (z_{y(i)} - z_{x(i)})^2 = \sum_{i=1}^N d_i (z_{y(i)} - z_{x(i)})^2 + \sum_{i=1}^N (1 - d_i) (z_{y(i)} - z_{x(i)})^2 = D_{nec} + D_{suf}$$

where $E\{Z_{y(i)} | \text{null XY association}\}$ is the expected value of the standardized outcome membership score for case i given a null association between the hypothesized cause and the outcome.¹⁵

With $Z_{x(i)}$ and $Z_{y(i)}$ normally distributed by definition, a null association implies independence of $Z_{y(i)}$ and $Z_{x(i)}$ and thus $E\{Z_{y(i)} | Z_{y(i)} \otimes Z_{x(i)}\} = E\{Z_{y(i)}\} = \bar{Z}_y$, where $Z_{y(i)} \otimes Z_{x(i)}$ indicates independence and \bar{Z}_y gives the sample mean of $Z_{y(i)}$. Thus, substituting \bar{Z}_y for $E\{Z_{y(i)} | \text{null XY association}\}$ gives the minimum-distance expected value

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Appendix 4: Pseudo Code for Programs for Csuff, Dsuff

- A. input the parameters that are scalars
Input the data as a rectangle without missing values.
- B. Label the permutations (ie X configurations), calculate fuzzy $X = \min(X_k)$ for each configuration, count the length of Y and the Number of instances in each X configuration (N in set for X where $Y_i > X_i$)
- C. Calculate Consistency for Sufficiency
Calculate Distance for Sufficiency
- D. Output plots of the X and Y as fuzzy scores
Output plots of the rescaled ZY by ZX, and a table of Csuff, Dsuff
- E. Test for sensitivity to the parameters by looping around, changing either the damping factor or the measurement error.

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SAMPLE DATA SETS:

1) the website of my course for some past years:

<http://Course-data.ccsr.ac.uk/gca>

2) the COMPASS web site (*sic*) www.compass.org (They have a lot of CSV files there)

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Appendix 4: Pseudo Code for Programs With Bootstrap

•Input S the scale of the bootstrap activity.
Start loop.

•Create S=1000 resamples with replacement

These have some repeats of cases.

Each case in each sample is a replica of the original data.

Some cases in the data may not appear, at random in a particular sample.

- A. input the parameters that are scalars
Input the data as a rectangle without missing values.
- B. Label the permutations (ie X configurations), calculate fuzzy $X = \min(X_k)$ for each configuration, count the length of Y and the Number of instances in each X configuration (N in set for X where $Y_i > X_i$)
- C. Calculate Consistency for Sufficiency
Calculate Distance for Sufficiency
- D. Output plots of the X and Y as fuzzy scores
Output plots of the rescaled ZY by ZX, and a table of Csuff, Dsuff
- E. Test for sensitivity to the parameters by looping around, changing either the damping factor or the measurement error.

End loop. Average the Csuff over all the S samples.

Average the Dsuff over all the S samples.

•Empirically compare the mean of Csuff with the original Csuff (Bias of consistency measure)

•Empirically compare the mean of Dsuff with the original Dsuff (Bias of distance measure)

•Create a table or graph showing the empirical distribution of the S Csuff's, 95% of which forms a credible interval.

•Create a table or graph showing the empirical distribution of the S Dsuff's, 95% of which forms a credible interval.

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