Quasi-variances and extensions

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Abstract: The notion of quasi-variances, as a device for both simplifying and enhancing the presentation of categorical-predictor effects in statistical models, was developed in Firth and de Menezes (*Biometrika*, 2004, 65–80). The approach generalizes the earlier ideas of Ridout (*GLIM Proceedings*, 1989) and of Easton, Peto and Babiker (*Statistics in Medicine*, 1991 — 'floating absolute risk', which has become rather controversial in epidemiology). In this talk I will outline and exemplify the method to show how it can be useful, and discuss its extension to some other contexts such as parameters that may be arbitrarily scaled and/or rotated.

Keywords: Floating absolute risk; model summary

1 Quasi-variances: The basic idea

When presenting the results of statistical modelling, one very standard summary is a table of parameter estimates and standard errors; in Bayesian analysis, an analogous device is a table of posterior means and standard deviations or — if space permits — a series of marginal views of the posterior density. The device of 'quasi-variances' aims to improve such summaries in situations where at least some of the parameters of interest relate to the effect of a categorical predictor variable. In such situations, *contrasts* among the parameters typically are identified and of interest. Most commonly the standard summary is based on an arbitrarily selected subset of contrasts, for example contrasts with the first or last level of a factor, or with an average over all of the levels. Such a summary works well for those specific contrasts, but does not facilitate valid inference on other contrasts not in the selected subset.

Quasi-variances overcome this difficulty as follows. (The exposition here will be in terms of estimates and standard errors; it could equally well be made in terms of posterior means and standard deviations.) For a set of parameters $\beta_1, \ldots, \beta_p$, we approximate the variance of any contrast $\sum c_r \hat{\beta}_r$ (where $\sum c_r = 0$) by $\sum c_r^2 q_r$, in which the quantities $q_1, \ldots, q_p$ are so-called *quasi-variances*. When good quasi-variances can be found — that is, when the approximation is reasonably accurate for all contrasts of potential interest — this yields a simple summary table from which valid approximate

inference can be drawn about *any* contrast. The simplicity stems from the fact that the $\{q_r\}$ can be read *as if* they were the variances of p uncorrelated estimates. This also allows for simple graphical presentations, for example with a point estimate and error bar for each parameter, whose 'Pythagorean' interpretation is both informative and familiar.
This basic idea was first suggested by Ridout (1989), in the context of estimates from a balanced experimental design. Easton, Peto and Babiker (1991) independently suggested it under the name 'floating absolute risk', with some particular epidemiological applications in mind. A further influential reference is Cox and Reid (2000, p237). In epidemiology the method has proved to be rather controversial (e.g., Easton and Peto, 2000, and references therein); this seems to be partly because the idea of Easton et al. (1991) was not always well enough understood, and partly because the specific approximation recipe used in Easton et al. (1991) was not ideal. Menezes (1999), Firth and Menezes (2004) and Plummer (2004) studied the approximation in detail and suggested methods that are more generally successful. The work of Ridout (1989), whose approximation recipe was indeed one of the 'generally successful' variety, was sadly unknown to the epidemiologists whose arguments about the method's merits spanned several subsequent years.

2 Aims in this talk

In this talk I will review why and when the method of quasi-variances works well, and I will discuss some examples of its fruitful application. The controversy surrounding 'floating absolute risk' will be demystified.
Attention will then turn to extensions of the method:

(i) To some less standard contexts where contrasts are still the identifiable parameter combinations of interest. These contexts include:

- Bradley-Terry models for binary 'tournaments' (Turner and Firth, 2010);
- the homogeneous RC(1) association model of Goodman (1979), for contingency tables;
- multinomial logit regression models for categorical-response data;
- certain other often-used multiplicative interaction models, such as the 'unidiff' model from social mobility studies (Erikson and Goldthorpe, 1992; Xie, 1992).

(ii) To some more general situations, where the contrasts of interest are identified only after fixing some other aspect of parameterization such as *scale* or *angle of rotation*. These include:

- the *non*-homogeneous Goodman RC(1) association model;

– the (homogeneous or non-homogeneous) Goodman RC(2) association models;
– some standard item-response models (Rasch-type scaling models);
– factor analysis of multivariate data.

3 Software

The *R* package qvcalc (Firth, 2003b) implements the basic method efficiently, with direct interfaces to various prominent classes of model object in *R*; summary capabilities include the routine reporting of the accuracy of computed quasi-variances, and facilities for readily interpreted 'error bar' plots of effects of interest. The same package also underlies a simple web-based calculator (originally developed using *Xlisp-Stat*; see Firth, 2000).

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