The Social Relations Model for count data

with an application to inter-household meat sharing in Nicaragua

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Data for first 10 (of 35) households

- # of gifts of meat and fish given by household *i* to household *j* over 8 months

Giver Household	Receiver Household 1	2	3	4	5	6	7	8	9	10
1	-	0	6	2	4	8	2	1	0	9
2	3	-	1	2	0	3	0	42	4	0
3	23	2	-	10	13	38	1	2	0	1
4	4	1	0	-	1	13	0	0	0	1
5	2	1	2	6	-	12	1	0	0	2
6	2	0	2	3	14	-	0	0	0	0
7	1	1	0	1	1	1	-	0	0	0
8	2	6	0	0	0	0	0	-	1	0
9	1	2	0	0	2	0	4	4	-	0
10	70	1	5	6	11	18	3	4	0	-

What is the Social Relations Model?

- The SRM (Kenny et al., 2006) is a conceptual model for dyadic data:
  - The SRM decomposes the relationship response variance into: giver-, receiver- and relationship-specific variance components
  - The SRM allows for the correlation of giver and receiver effects
  - The SRM allows for correlation of responses within dyads
- The SRM is traditionally calculated using ANOVA or estimated as an SEM
  - Multilevel approach is very rarely used despite offering interesting extensions
  - The response data are almost universally treated as continuous

The SRM for continuous data

$$\mathbf{gifts}_{ij} = \beta_0 + g_i + r_j + e_{ij}$$

$$var(g_i) = \sigma_g^2, \qquad var(r_j) = \sigma_r^2, \qquad var(e_{ij}) = \sigma_e^2$$

$$corr(g_i, r_i) = \rho_{gr}, \qquad corr(e_{ij}, e_{ji}) = \rho_{ee}$$

where:

β_0 is the population mean
g_i is the giver main effect;
r_j is the receiver main effect;
e_{ij} is the unique relationship effect.

σ_g^2 is the between giver variance
σ_r^2 is the between receiver variance
σ_e^2 is the unique relationship variance
ρ_{gr} is the “generalised reciprocity” correlation
ρ_{ee} is the “dyadic reciprocity” correlation

The SRM formulated as a multilevel model

- The SRM can be formulated as a bivariate response cross-classified multilevel model:

$$\textbf{gifts}_{ij} = \beta_0 + g_i + r_j + e_{ij}$$
$$\textbf{gifts}_{ji} = \beta_0 + g_j + r_i + e_{ji}$$
$$\begin{pmatrix} g_i \\ r_i \end{pmatrix} \sim \mathrm{N}\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_g^2 & \\ \sigma_{gr} & \sigma_r^2 \end{pmatrix}\right\}$$
$$\begin{pmatrix} e_{ij} \\ e_{ji} \end{pmatrix} \sim \mathrm{N}\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & \\ \sigma_{ee} & \sigma_e^2 \end{pmatrix}\right\}$$

where:

- Relationships are nested within the cross-classification of givers by receivers
- The fixed and random parameters are constrained across equations

The SRM is a particularly interesting multilevel model

- The SRM has unusual features:

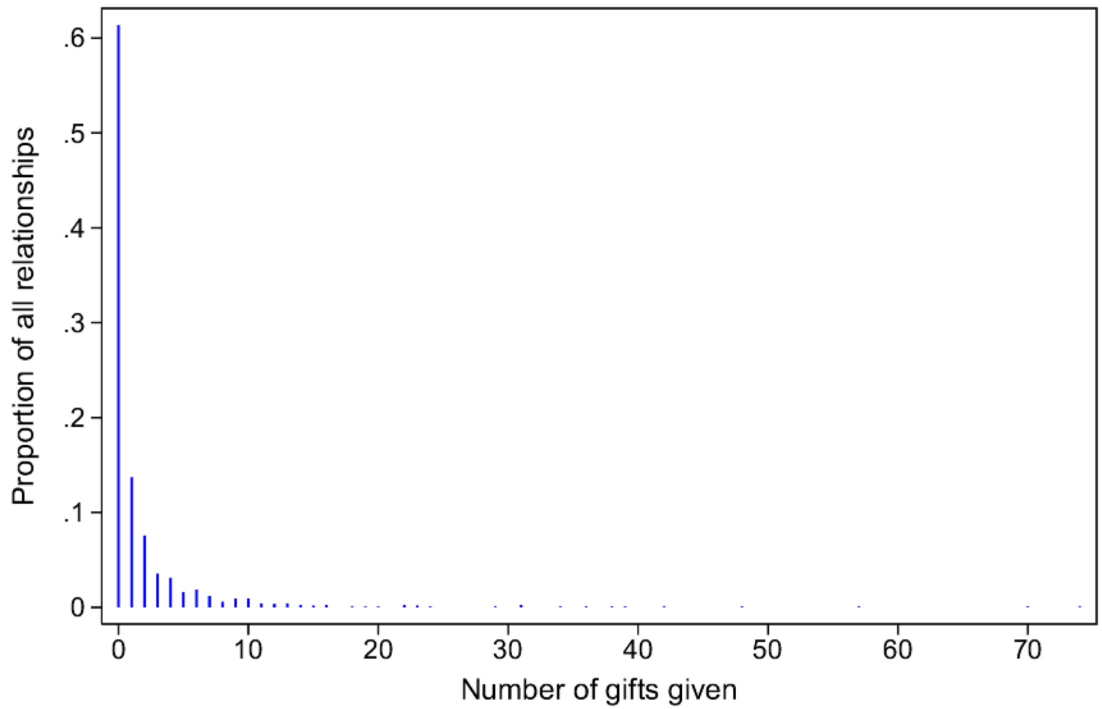
  1. Correlated random classifications
     The generalized reciprocity correlation allows the giver and receiver random effects to be correlated

  2. Random interaction classification
     The dyadic reciprocity correlation allows the giver and receiver random effects to be non-additive

     - Some dyads give more than others even after controlling for giver and receiver main effects

     - A re-parameterisation of the model explicitly reveals these random dyad effects (i.e. a random interaction classification)

$\mathbf{gifts}_{ij}$ is a count variable



- Furthermore, not all households were present for the full 8 months and so the exposure varies across dyads

The SRM for count data

- The SRM model for count data:

$$\mathbf{gifts}_{ij} \mid \boldsymbol{\mu}_{ij} \sim \text{Poisson}(\boldsymbol{\mu}_{ij})$$
$$\mathbf{gifts}_{ji} \mid \boldsymbol{\mu}_{ji} \sim \text{Poisson}(\boldsymbol{\mu}_{ji})$$

$$\ln(\boldsymbol{\mu}_{ij}) = \ln(\mathbf{months}_{ij}) + \beta_0 + g_i + r_j + e_{ij}$$
$$\ln(\boldsymbol{\mu}_{ji}) = \ln(\mathbf{months}_{ji}) + \beta_0 + g_j + r_i + e_{ji}$$

$$\begin{pmatrix} g_i \\ r_i \end{pmatrix} \sim \text{N}\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_g^2 & \\ \sigma_{gr} & \sigma_r^2 \end{pmatrix}\right\}$$
$$\begin{pmatrix} e_{ij} \\ e_{ji} \end{pmatrix} \sim \text{N}\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & \\ \sigma_{ee} & \sigma_e^2 \end{pmatrix}\right\}$$

- The SRM model for other types of data can be specified by changing the distribution and link functions

Fitting the SRM in multilevel software: Classical estimation

- Snijders and Kenny (1999) show how the SRM can be fitted to continuous data using IGLS in MLwiN
  - They follow Rasbash and Goldstein (1994) and specify the cross-classified model as a constrained hierarchical model
  - In our application, this results in 35 giver and 35 receiver random effects and 105 associated (co)variance parameters
  - 102 constraints ($= 3 \times (35 - 1)$) are required to identify the model
  - One further constraint is required for the level 1 variances

- The S&K approach can not be used for discrete data using IGLS in MLwiN
  - The quasilikelihood methods available in MLwiN are biased
  - The large number of constraints also cause MLwiN also to fall over

- The S&K approach would also not work using quadrature (say in Stata)
  - The large number of random effects make quadrature computationally infeasible

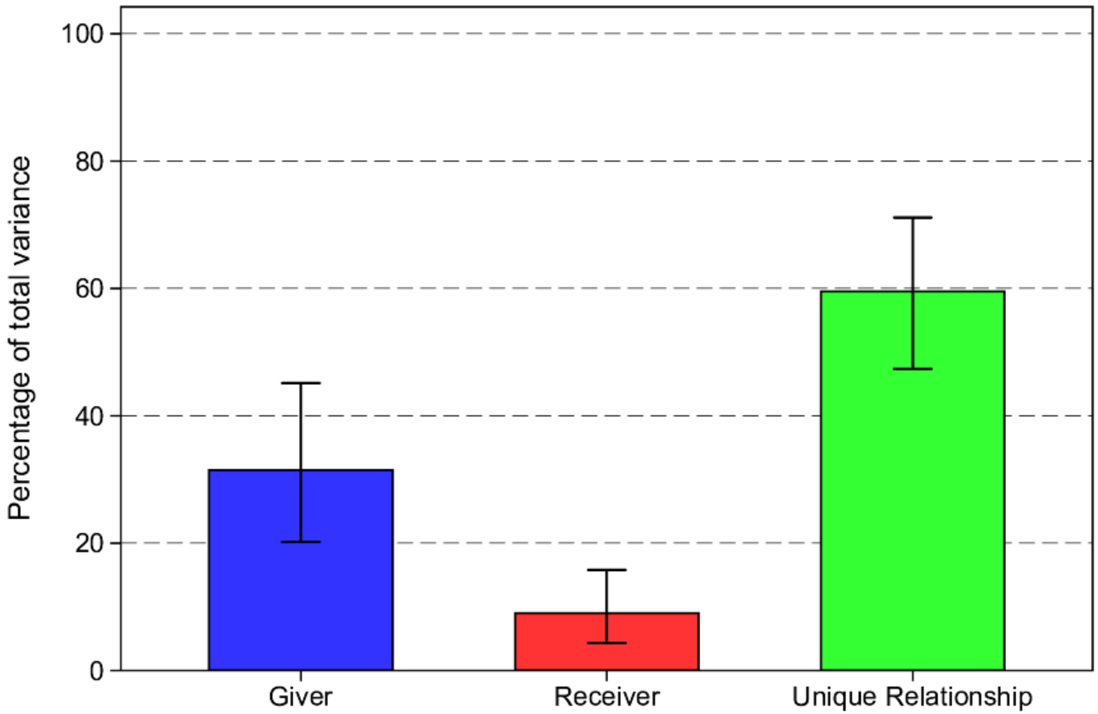
Fitting the SRM in multilevel software: Bayesian estimation

- Cross-classified models can be fitted easily using MCMC

- MCMC in MLwiN
  - Not possible to correlate the giver and receiver random effects
  - Not possible to constrain for the relationship variances to be equal, although a re-parameteristion exists if the dyadic reciprocity is positive
  - Thus, MCMC in MLwiN can only be used if generalized reciprocity is assumed to be zero and the dyadic reciprocity is assumed to be positive

- MCMC in WinBUGS
  - Can specify and fit the SRM for count data!
  - Computationally slow

Influences on household giving



- Generalised reciprocity correlation = 0.167 (-0.225, 0.537)
- Dyadic reciprocity correlation = 0.962 (0.915, 0.995)

Predictor variables

- Household level characteristics entered separately for giver and receiver households:
  - Age of male head of household
  - Household own harvest of meat and fish

- Dyad level characteristics:
  - Distance between households
  - Genetic relatedness of households

- Directed relationship level characteristics:
  - Mother to child relationship different from child to mother

Conclusions

- The SRM for count data is a simple extension to the standard SRM
  - In this sense it is strange that it has not been used
- However, it cannot easily be fitted by classical methods in multilevel software
  - MCMC in WinBUGS
  - MCMC in Stat-JR
- The SRM only accounts for dyadic interdependence
  - Contrast with social network statistical models which allow for triadic interdependence
- Count relationship data will typically suffer from excess zeros
  - Zero inflated model (i.e. mixture model)

References

- Kenny, D. A., Kashy, D. A. and Cook, W. L. (2006) *Dyadic data analysis*. New York: Guilford Press.

- Koster, J. M. (forthcoming) Inter-household meat sharing among Mayangna and Miskito horticulturalists in Nicaragua. *Human Nature*.

- Rasbash, J. and Goldstein, H. (1994) Efficient analysis of mixed hierarchical and crossed random structures using a multilevel model. *Journal of Behavioral Statistics*, 19, 337-350.

- Snijders, T.A.B., and Kenny, D., (1999) The Social Relations Models for family data: A multilevel approach. *Personal Relationships*, 6, 471-486.