Department of Quantitative Social Science

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John Micklewright
Sylke V. Schnepf
Pedro N. Silva

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Department of Quantitative Social Science. Institute of Education, University of London. 20 Bedford way, London WC1H 0AL, UK.

Peer effects and measurement error: the impact of sampling variation in school survey data

John Micklewright[*], **Sylke V. Schnepf**[†], **Pedro N. Silva**[‡§]

Abstract. Investigation of peer effects on achievement with sample survey data on schools may mean that only a random sample of peers is observed for each individual. This generates classical measurement error in peer variables, resulting in the estimated peer group effects in a regression model being biased towards zero under OLS model fitting. We investigate the problem using survey data for England from the Programme for International Student Assessment (PISA) linked to administrative microdata recording information for each PISA sample member's entire year cohort. We calculate a peer group measure based on these complete data and compare its use with a variable based on peers in just the PISA sample. The estimated attenuation bias in peer effect estimates based on the PISA data alone is substantial.

JEL classification: C21, C81, I21.

Keywords: peer effects, measurement error, school surveys, sampling variation.

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[*] Institute of Education, University of London. E-mail: J.Micklewright@ioe.ac.uk

[†] School of Social Sciences and Southampton Statistical Sciences Research Institute, University of Southampton. E-mail: svs@soton.ac.uk

[‡] Instituto Brasileiro de Geografia e Estatística and Southampton Statistical Sciences Research Institute, University of Southampton. E-mail: P.L.N.Silva@soton.ac.uk

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1. Introduction

A major strand of the literature on peer group effects is concerned with the definition of peers and the measurement of their attributes. Hanushek et al (2003) argue that issues of omitted and mismeasured variables are probably more important than those surrounding the simultaneous determination of peer interactions – the 'reflection problem' discussed by Manksi (1993) that has seen much attention. We address an aspect of peer group measurement that often arises in analyses based on sample survey data, but which is also often ignored. If the survey's design means that only a random sample of peers is observed for each indvidual, rather than all peers, then any summary statistic of peer attributes that is based on the survey data and used as an explanatory variable is subject to sampling variation. This generates measurement error similar in form to the textbook case of errors-in-variables. As a result, the estimated peer group coefficient in an OLS regression is biased towards zero.

The problem has been recognised, for example by Ammermüller and Pischke (2009) for whom sampling variation is one source of error in peer group measurement. There is also a parallel literature in statistics that is concerned with multilevel models applied to survey data with a hierarchical structure when measures of variables at a higher level are formed by averaging the characteristics of units at a lower level (Woodhouse et al 1996, Kravdal 2006). These papers have warned of the consequences of sampling variation in peer averages, but have been unable to conclude categorically about the extent of bias in any particular empirical setting. As Ammermüller and Pischke (2009) note, the bias will depend inter alia on the relative sizes of the within- and between-group variation in the individual characteristics. The bias is greatest when the former dominates – sampling from relatively heterogeneous groups can result in large sampling error.

We are able to quantify the extent of the bias in peer group estimates obtained with school survey data since we have information on the population from which each sample of peers in the survey is drawn. We compare the OLS estimate of the peer group parameter when the peer average is calculated with the survey sample of peers with the OLS estimate obtained when the average is calculated for the population peer set. We do this for one country, England, in a major international school survey, the Programme for International Student Assessment (PISA), which measures the cognitive achievement of 15 year olds. The international reports on PISA emphasise the estimated impact of peers on cognitive achievement e.g. OECD (2001, chapter 8), OECD (2007, chapter 5). Subsequent papers have also estimated peer effects with the data e.g. Fertig (2003), Schindler-Rangvid (2003), Entorf and Lauk (2006), Schneeweis and Winter-Ebmer (2007). The potential impact of sampling variation on peer measurement in the PISA data has not been highlighted.

Section 2 relates the classical measurement error problem to the PISA survey design. Section 3 describes our PISA data for England, which comprise the achieved sample in 2003 of responding schools and pupils together with data from administrative registers on all 15 year-olds in the sampled schools. Section 4 presents results from regressions for cognitive achievement in which peer variables are calculated with the relevant group defined in three different ways: the population of all other 15 year olds in the individual's school, the selected sample of other pupils in the school, and the responding sample of other pupils. Section 5 concludes.

2. Classical measurement error and the PISA sample design

In a regression model with one explanatory variable, classical measurement error in that variable leads to bias towards zero in the OLS estimate of the slope parameter – the 'iron law of econometrics' (Hausman 2001). The size of this attenuation bias is determined by the relative magnitudes of the variances of the unobserved true variable x_i and of the observed explanatory variable z_i. Let:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{1}$$

be the target regression model, where y_i is the response (measured without error) and ε_i is the error (disturbance) term.

Under a classical measurement error scenario, the observed values of the predictor variable z_i are related to the true unobserved values x_i as follows:

$$z_i = x_i + u_i \tag{2}$$

Therefore the researcher is forced to estimate:

$$y_i = \beta_0 + \beta_1 z_i + (-\beta_1 u_i + \varepsilon_i) \tag{3}$$

The OLS estimator of the slope coefficient for the observed data is given by:

$$\hat{\beta}_{1OLS} = \frac{(n-1)^{-1}\sum_i (z_i - \overline{z})(y_i - \overline{y})}{(n-1)^{-1}\sum_i (z_i - \overline{z})^2} = \frac{\mathrm{cov}(z_i, y_i)}{\mathrm{var}(z_i)} \tag{4}$$

Under the standard assumption that

$$(x_i, u_i, \varepsilon_i) \sim MN\left[(\mu_x, 0, 0); diag(\sigma_x^2, \sigma_e^2, \sigma_u^2)\right]$$

are independent random vectors with a common Multivariate Normal distribution, it follows that, see e.g. Fuller (1987):

$$E(\hat{\beta}_{1OLS}) = \frac{VAR(x)}{VAR(z)}\beta_1 = \frac{\sigma_x^2}{\sigma_z^2}\beta_1 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}\beta_1 \tag{5}$$

Under slightly weaker assumptions the following result holds for large samples:

$$\text{plim}(\hat{\beta}_{1OLS}) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}\beta_1 \tag{6}$$

Thus measurement error implies that the composite error term in brackets in (3) is negatively correlated with the observed z_i, leading to bias in $\hat{\beta}_{1OLS}$, the OLS estimator of the slope β_1 in the target model (1).

Now assume that the target regression model includes additional explanatory variables, $\mathbf{t}_i$, free of measurement error:

$$y_i = \beta_0 + \beta_1 x_i + \boldsymbol{\beta}_2'\mathbf{t}_i + \varepsilon_i \tag{7}$$

The textbook result is that the OLS estimate of β_1 based on the observed covariate z_i is still biased towards zero. The OLS estimate of $\boldsymbol{\beta}_2$ is also biased but in unknown directions (e.g. Greene 1993: 280-4), the measurement error in one variable contaminating the estimates of the other parameters in the model.

Suppose that y_i represents an individual's test score and x_i represents a measure of an individual's peer group, defined as the average value of a characteristic for all other persons in the individual's age cohort at school. This is a broad definition of peers, adopted in many studies out of necessity, for example Hanushek et al (2003), although authors often recognise a narrower definition such as the class may be more suitable. Many school surveys have a sampling design that results in x_i being measured with error, since only a random sample of pupils is selected within each school for inclusion in the survey rather than all pupils.

This problem is shared by PISA. The survey has a two-stage design. Schools are sampled with probability proportional to size and then 35 pupils aged 15 are randomly sampled within each school. In England in 2003, the 35 students were sampled out of what is an average of about 170 students of this age per school. The mean characteristics of an individual's schoolmates that are observed in the PISA sample will be measured with error by z_i.[1] The error, $e_i = z_i - x_i$ is the result of sampling variation. Some of its properties resemble

[1] The only exception, where x is observed, will be for small schools with 35 or less 15 year olds since in this case all students of this age in the school are sampled by PISA.

those of textbook 'classical' measurement error defined above, u_i. Critically, $COV(e_i, x_i)$ should be close to zero. On the other hand, $CORR(e_i, e_j)$ will be very high for students in the same school, although it should again be zero for students in different schools. In the next section we investigate these features in practice in the PISA data.[2]

3. The PISA data for 2003 in England and the measurement of peer variables

The 2003 PISA round in England resulted in data being collected from pupils at 159 responding schools. PISA tests 15 year olds on their competence in maths, science and reading. In 2003 maths was the 'major' subject to which the most time was devoted in the test instruments, while science and reading were 'minor' subjects, with less test time.

We have access to a version of the survey data that links schools and pupils to administrative registers containing information for all 15 year old pupils in the country. These registers provide us with one measure of pupils' socio-economic status, namely an indicator for whether they receive Free School Meals (FSM) – a state benefit for low income families. This is the standard focus for research into social background in England's schools based on administrative data e.g. Burgess et al (2004), Goldstein and Noden (2003). A similar variable is used in US research on peer effects based on administrative registers and in that context has been summarised as 'likely to be a noisy measure of peer economic circumstances' (Hanushek et al 2003: 537) that may 'proxy omitted or mismeasured factors that affect individual achievement, leading to biased results that quite generally exaggerate the importance of peers' (ibid: 530). The same is true in the UK. However, our ambition is not to estimate the 'true' impact of peers. Rather it is to demonstrate the impact of measurement error bias resulting from survey design, albeit on the estimated parameter of an imperfect indicator of peer characteristics. By analogy, measurement error bias will be present in parameter estimates of other peer indicators based on richer family background variables collected in the survey data but which are not present in the administrative registers. In the terminology of Manski (1993), peer receipt of FSM allows us to estimate exogenous 'contextual' peer group effects.

[2] Not all school surveys share this problem. In the Trends in International Maths and Science Study (TIMSS), for example, a whole class is randomly selected within each school. If the peer group is defined as the whole class rather than the whole cohort, then all peers are observed. Toma and Zimmer (2000) investigate peer effects with TIMSS data.

We estimate regression models for 3,459 responding pupils in state schools for whom we have information on FSM receipt. We exclude children at private schools for whom this information is not recorded (receipt is likely to be zero in this group) and a small number of respondents in state schools for whom the information is also missing (these two groups represent 5.9 percent and 1.5 percent respectively of all responding pupils). Among the state school pupils that we analyse, 10.4 percent received FSM. We take the proportion of other 15 year olds in each individual's school who receive FSM as our measure of the peer group composition. The true value, x_i, is measured by receipt of FSM among all other 15 year olds in the individual's school, while the 'observed' value in the PISA survey data, z_i, is measured by receipt of FSM among the 35 sampled students, less the individual concerned, in the individual's school. Measurement error e_i is given by z_i minus x_i.

A complication is introduced by non-response to the survey; 23 percent of sampled pupils in England in PISA 2003 declined to participate in the survey.[3] This means that we can define the peer measure based on the survey data in two ways: (i) students sampled for PISA and, (ii) the subset of responding students. In the first case, z_i is indeed based on the 35 sampled students in each school, less the individual concerned. Here the measurement error e_i reflects only sampling error. In the second case, z_i is based on the other responding students in each individual's school. Here e_i is affected in addition by the pattern of response.

Figure 1 plots the observed z_i against the true x_i, where z_i in this case is defined in the first of the ways just described. The two measures are strongly correlated but there is also a fair degree of scatter around the 45 degree line reflecting the impact of sampling error. The extent of the sampling error, $e_i = z_i - x_i$, is shown more directly in Figure 2. The error averages close to zero but ranges from about –0.2 to +0.2. The standard deviation of 0.058 may be compared with the mean of the true x_i, 0.118. The extent of the sampling error is sufficient for us to expect that a non-trivial degree of bias will arise from the use of the survey-based measure of the peer variable.

The properties of the observed e_i are not identical to those of u_i in the textbook measurement error set-up described earlier in the section. We have already noted that the correlation of sampling errors in peer measurement will be very high for students in the same school. In practice, it is also the case that we observe a correlation between e_i and true peer value x_i of –0.18, rather than the value of zero in the text-book case. (We easily reject the hypothesis that the correlation is zero e.g. at the 0.1 percent significance level.) Figure 3 plots

[3] See Micklewright, Schnepf, and Skinner (2010) for details.

the two variables against each other. There is a bounding of both x_i and z_i from below by zero. While true x_i is only zero for one school, measured z_i is zero for about 10 percent of our pupil sample: sampling from schools with low levels of FSM can often result in there being no peers in the PISA sample who are in receipt (recall that on average only 10 per cent of pupils receive the benefit). In this case $e_i = -x_i$ and these are the observations on the line running from north-west to south-east at the left hand side of the graph. But the negative correlation is still present with these observations excluded.

Ignoring these differences, if we were to approximate u_i by e_i in equation (3), and also ignore that we will be estimating a regression with more than one explanatory variable, we would conclude from the textbook formula (6) that the plim of the OLS estimate of the coefficient on the peer measure differs from the true value by a factor of 0.72 when measuring peer FSM receipt using the survey data.

4. Estimated bias in the peer group and other coefficients

Table 1 shows the results of estimating a linear regression model of the PISA maths test score for state school pupils in England. Results for the FSM peer measure are very similar using either the science or reading test scores as an alternative. The maths score has a weighted mean of 501.8 and a weighted standard deviation of 87.8. We also apply weights to the data when estimating our regression models.[4]

Besides a binary variable indicating own receipt of FSM and a continuous variable measuring the proportion of peers receiving FSM, we include a number of controls: dummy variables indicating gender, the level of the mother's education, and the number of books in the home. There is no measure of family income in PISA, so the FSM dummy is the only direct indicator of low income available to us. Mother's education is a well-recognised correlate of children's educational attainment, e.g. Haveman and Wolfe (1995). The association reflects both a direct impact on the quantity and quality of time and goods inputs in the child and an indirect impact coming through family income. It may also proxy unobserved parental ability that is passed on to the child through his or her gene endowment. The number of books in the home is estimated by the child and reported in categorical form. This is a standard variable collected in international surveys of children's learning and is often

[4] The weights are those supplied with the data by the OECD. They adjust for different sampling probabilities, the level and pattern of school response, and the level of pupil response. See Micklewright and Schnepf (2006) for details. Point estimates and hence our estimates of attenuation bias are very similar if we use unweighted data.

used to proxy family background. It is used as the main measure of both individual and peer characteristics in the analysis of peer effects by Ammermüller and Pischke (2009).[5]

Column 1 gives results when we measure peer FSM receipt with the true value, x_i, the proportion of all other 15 year olds in the individual's school who are receiving this benefit. The estimated coefficient is well determined. It implies that a one standard deviation rise in peer FSM receipt, equal to 0.058, is associated with a fall of about 0.15 of a standard deviation of the maths score. This is quite large, above the range of most peer effect sizes measured in this standardised way that are reported by Ammermüller and Pischke (2009) from their review of the literature, but it is similar to the size of the average effect these authors find for the six European countries in their study of primary school children.

There is no measurement error bias resulting from sampling in the peer effect estimate in column 1 (although there may be omitted variable bias due to peer FSM proxying other unobserved factors influencing the maths score). This is in contrast to the estimate of the peer effect reported in columns 2 and 3, obtained by using measures of FSM receipt based respectively on those peers drawn for the PISA sample and on the subset who respond. A comparison with the figure in column 1 gives an estimate of the extent of attenuation bias present in the estimates obtained in columns 2 and 3. The estimated coefficients in columns 2 and 3 are roughly half that in column 1, indicating a rather larger problem in practice than would be suggested by the calculation we reported at the end of the last section based on the textbook formula for the extent of attenuation bias in a regression with a single explanatory variable.

Measurement error in one of the explanatory variables in a regression model also biases estimates of the coefficients of the other variables and we see evidence of this when comparing the other parameter estimates in columns 2 and 3 with those in column 1. Moving to the FSM peer measures based only on sampled or responding pupils tends to lead in this case to estimates that are biased upwards in absolute size, rather than attenuated as in the case of the coefficient on the peer measure itself. Coefficients on several variables rise in absolute size by an amount equal to about one to one and a half standard errors: the individual FSM receipt dummy, the mother's secondary education dummy (the coefficient doubles in this case) and the books dummy.

To investigate the robustness of these results we estimated the model with other specifications. We first added further variables to measure socio-economic background,

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[5] See also Schütz et al (2008) for a spirited defence of the use of books in the home as a measure of socio-economic background.

including dummies for the father's as well as the mother's education and including a more detailed set of dummies for the number of books in the home. In each case we calculated an estimated attenuation ratio by dividing the coefficient on the peer FSM receipt when measured with sampled or responding peers by the coefficient on the 'true' FSM peer variable reported in column 1 of Table 1. The values were essentially unchanged from those implied by the results in Table 1 – around 0.5. And, as before, the coefficients on other variables in the model, including the new ones, tended to be biased upwards in absolute terms when using the peer FSM variables based on sampled or responding peers. The implied adjustment factor for measurement error that would need to be applied to the peer coefficients in columns 2 and 3 is therefore about 2.0. We then experimented with models using the basic specification shown in Table 1 in which we also included the peer values of the four other principal explanatory variables: gender, mother's secondary education, mother's tertiary education and the books dummy. In the case of these new peer variables, we can only measure the characteristics of responding peers in the PISA sample – we do not have information on the peer values for all 15 year olds in each individual's school. The analogue of the model in column 1 is therefore slightly difficult to interpret – the peer variable for FSM refers to the 'true' value based on the population of all 15 year olds while the other peer variables are based only on PISA respondents. The bias in this case for the estimated peer FSM coefficient in the analogues of columns 2 and 3 was even larger with an estimated attenuation ratio of about 0.35, although for the reason explained we have more confidence in the estimates obtained with the specification reported in Table 1.[6]

5. Conclusions

We have investigated attenuation bias in peer effect estimates that arise when information is available for just a random sample of peers rather than all peers, a situation that is not uncommon in school surveys. In our particular empirical setting of the PISA sample for England for 2003 and a peer variable measuring the proportion of children receiving an in-kind benefit for low income families, we were able to exploit linked administrative data on benefit receipt among all children in the same age cohort at each individual's school. We found substantial attenuation bias in the estimated peer effect when measuring peer receipt

[6] Having checked diagnostics for multicollinearity with a focus on the Variance Inflation Factor, we did not include in the model variables indicating the proportion of peers for whom information on maternal education or the number of books is missing.

using just the peers present in the survey data. Biases were also present in estimates of other parameters.

These results suggest that caution is needed when estimating peer effects with survey data of the type we have used here. The extent of attenuation bias will of course vary with the empirical setting.[7] As far as use of PISA data is concerned, ceteris paribus one would expect to find less attenuation bias in countries where schools are more socially segregated (see Jenkins et al 2008), that is where between-school variation in pupil characteristics is high.

[7] In Silva et al (2010) we cast doubt on a simple adjustment factor for attenuation bias resulting from sampling error in the peer measure that is suggested in Neidell and Waldfogel (2008), who drew on Ammermueller and Pischke (2006, 2009).

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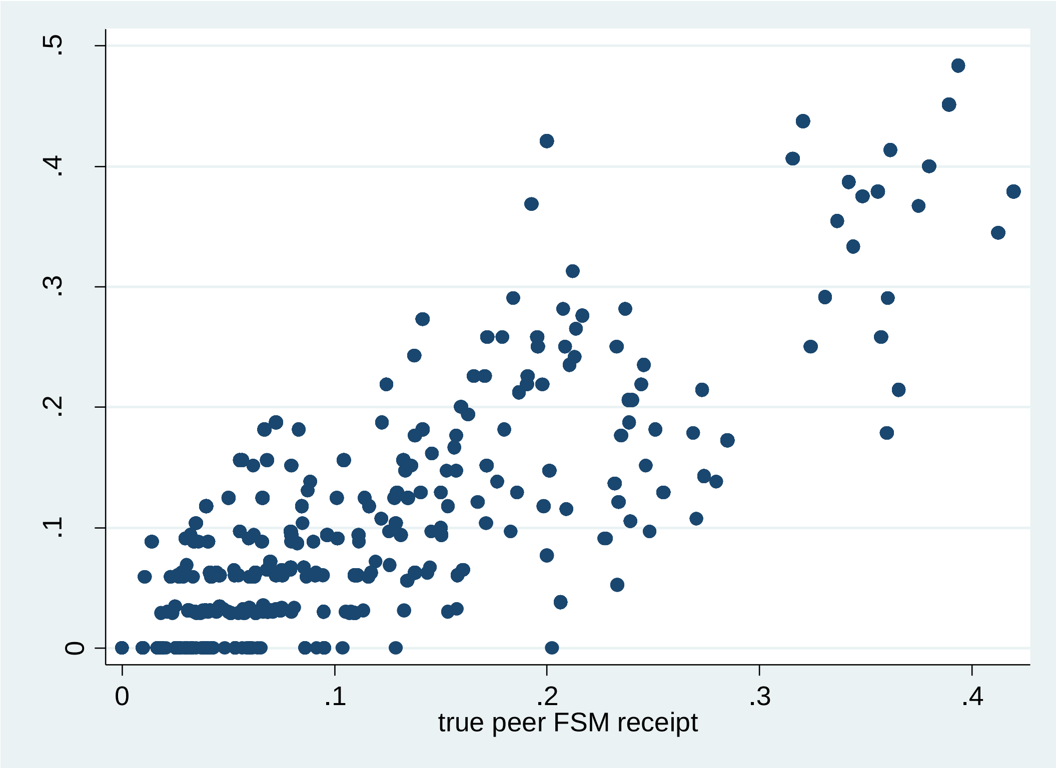
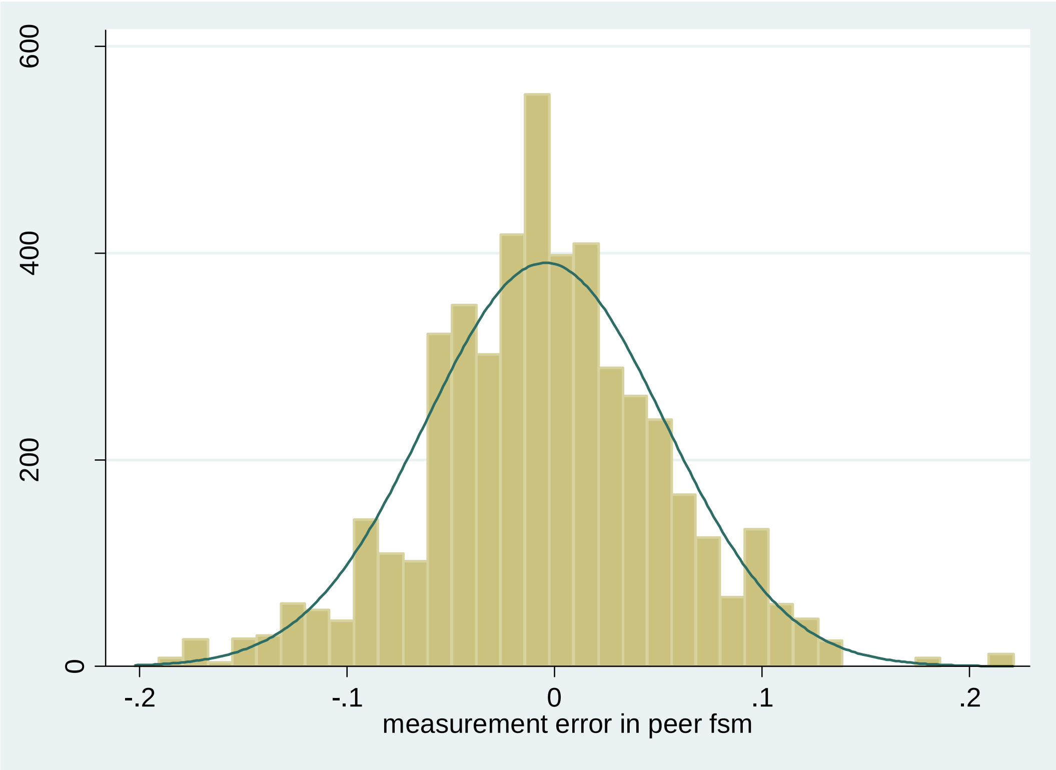
Figure 1. 'Observed' (z_i) and 'true' (x_i) peer FSM variables



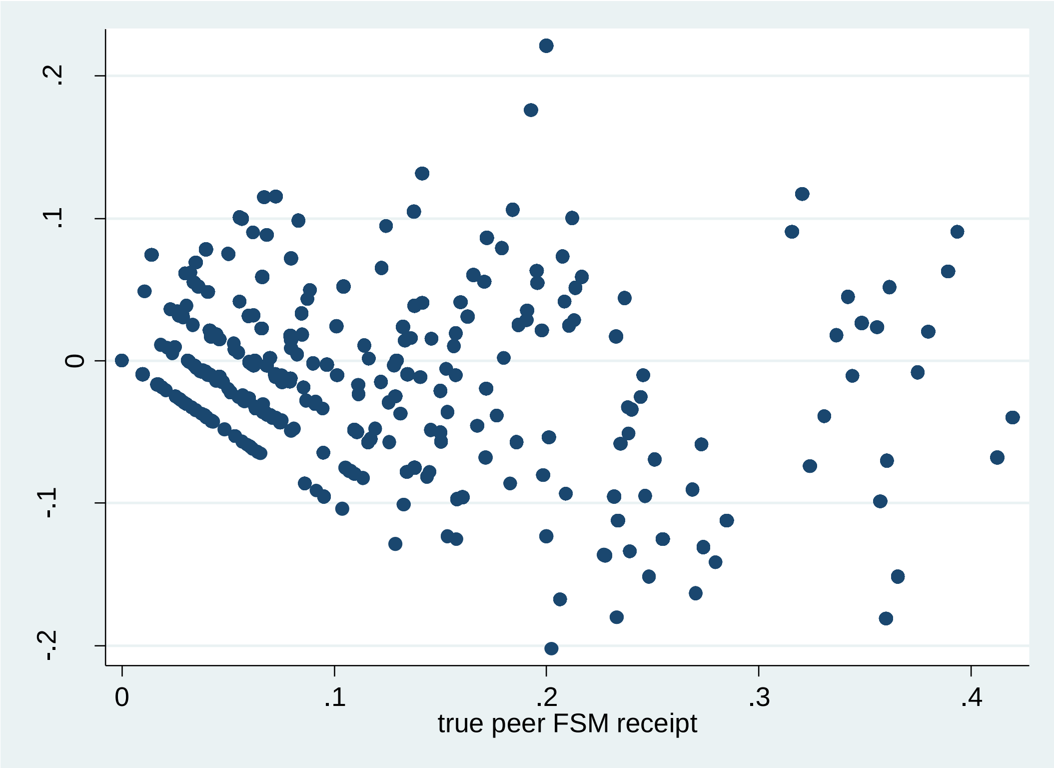
Notes. The observed peer FSM receipt relates to all sampled peers. The correlation coefficient is r=0.82.

Figure 2. Distribution of sampling error (e_i) in peer FSM variable



Notes: The graph shows $e_i = z_i - x_i$ with a normal distribution with the same mean and standard deviation superimposed. The observed peer FSM receipt, z_i, relates to all sampled peers. The mean and standard deviation of e_i are -0.004 and 0.058 respectively.

Figure 3. Sampling error (e_i) in peer FSM measure and 'true' (x_i) peer FSM



Notes: see notes to Figure 2.

Table 1. Estimates of regression models of PISA maths score

	1.	2.	3.
FSM in receipt (pupil)	-17.55	-23.42	-24.67
	(4.03)	(4.08)	(4.07)
True peer FSM receipt, *x*	-230.78		
	(30.95)		
Observed peer FSM receipt, *z* (sample)		-120.25	
		(26.65)	
Observed peer FSM receipt, *z* (responding pupils)			-109.06
			(25.39)
Female	-8.94	-9.14	-9.05
	(3.74)	(3.99)	(4.02)
Mother has secondary education	4.72	9.20	9.93
	(3.92)	(4.22)	(4.36)
Mother has teritary education	16.18	17.18	17.19
	(3.30)	(3.37)	(3.36)
Missing value mother secondary education	-24.24	-21.97	-21.34
	(6.05)	(6.34)	(6.45)
Missing value mother tertiary education	65.64	74.16	75.79
	(19.01)	(21.59)	(21.99)
More than 100 books at home	43.88	47.68	48.26
	(3.21)	(3.59)	(3.65)
Missing value books	-13.32	-16.84	-17.38
	(14.01)	(15.04)	(15.15)
Constant	510.29	492.01	488.86
	(6.15)	(6.08)	(6.14)
Observations	3,459	3,459	3,459
R-squared	0.21	0.17	0.17

Note: Standard errors in parentheses. Clustering in schools is allowed for when estimating the standard errors. Weighted data. Mean score = 501.8, SD = 87.8.