Path analysis for discrete variables:
The role of education in social mobility

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Outline

- Example: Analysis of social mobility
- Reminder: Linear path analysis
- Path analysis for general variables: definition
- Estimation of the effects and their standard errors
- Interpretation of the effects in the path analysis
  - in causal terms
  - in non-causal terms
- Example: Analysis of UK mobility data

Five variables will be considered today:

Social class:

- **Origin class** \((O)\): Person’s father’s class
- **Destination class** \((D)\): Person’s own class

...classified using a 3-class version of the Goldthorpe class schema:

- “Salariat” \((S)\)
- “Intermediate” \((I)\)
- “Working” \((W)\)

Education \((E)\), with seven ordered levels

+ Analysis stratified by **Sex** and **Period**
Today’s data

- Data from the British General Household Survey (GHS), as used by Goldthorpe and Mills (2004; in Breen (ed.), *Social Mobility in Europe*)
- Consider separately men and women, from the 1973 and 1992 surveys
- Respondents aged 25–59
- Sample sizes:

<table>
<thead>
<tr>
<th>Year</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>6276</td>
<td>6882</td>
</tr>
<tr>
<td>1992</td>
<td>4835</td>
<td>5284</td>
</tr>
</tbody>
</table>
Distributions of $D$ given $O$: Mobility tables

Example: Women in the 1992 survey

<table>
<thead>
<tr>
<th>Origin</th>
<th>Sal.</th>
<th>Int.</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salariat</td>
<td>759</td>
<td>508</td>
<td>228</td>
</tr>
<tr>
<td>Intermediate</td>
<td>519</td>
<td>503</td>
<td>342</td>
</tr>
<tr>
<td>Working</td>
<td>558</td>
<td>893</td>
<td>974</td>
</tr>
</tbody>
</table>
Associations of $O$ and $D$: Odds ratios

- For example, the 3 “diagonal” (log) odds ratios:

\[
\begin{array}{ccc}
 & S & I & W \\
S & O & O & \\
I & O & O & \\
W & & & \\
\end{array}
\]

- E.g. “I–S” odds ratio calculated from frequencies in cells $O$
- “W–I” and “W–S” associations similarly
Diagonal log odds ratios in the GHS data

<table>
<thead>
<tr>
<th></th>
<th>1973</th>
<th></th>
<th>1992</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
<td>Men</td>
<td>Women</td>
</tr>
<tr>
<td>I-S</td>
<td>.87</td>
<td>.42</td>
<td>.95</td>
<td>.37</td>
</tr>
<tr>
<td>W-I</td>
<td>.74</td>
<td>.65</td>
<td>.74</td>
<td>.47</td>
</tr>
<tr>
<td>W-S</td>
<td>2.00</td>
<td>2.19</td>
<td>1.85</td>
<td>1.76</td>
</tr>
</tbody>
</table>

Methodology Institute
Path analysis of social mobility

- Association between $O$ and $D$ describes (lack of) social mobility between generations
- This is the “total effect” of $O$ on $D$ discussed below
- Try to partition the total effect into...
  - **Indirect effect** $O \rightarrow E \rightarrow D$
    - $O \rightarrow E$: Class inequalities in educational attainment (and opportunity?)
    - $E \rightarrow D$: Dependence of class position on educational qualifications
  - **Direct effect** $O \rightarrow D$ not via $E$
    - Class inequalities in social networks, living conditions, social capital?
- How to assess relative sizes of these?
  - In particular, is the indirect effect dominant, as has been claimed in UK?
Path analysis of social mobility

In pictures:

O -> D

...elaborated into...

(Origin class, X) -> E -> (Destination class, Y)

(Education, Z)
Reminder: Linear path analysis

\[
E(Y|X, Z) = \beta_0 + \beta_x X + \beta_z Z \\
E(Z|X) = \alpha_0 + \alpha_x X
\]

\[
E(Y|X) = \int E(Y|X, Z) p(Z|X) \, dZ = \beta_0^* + \beta_x^* X
\]

where

\[
\beta_x^* = \beta_x + \beta_z \alpha_x
\]

i.e.

Total effect = Direct effect + Indirect effect
Path analysis for discrete variables

- How to define and estimate direct and indirect effects when $Z$ and/or $Y$ are categorical variables, and modelled as such?
- Here, *multinomial logistic models* for both
  - Education given Origin ($Z$ given $X$)
  - Destination given Origin and Education ($Y$ given $X$ and $Z$)
(Re)defining the effects for non-linear models

- Let $Y_l$ be an indicator for $Y = l$
  - Thus $E(Y_l) = P(Y = l)$
- Consider (any) two values $X_1$ and $X_2$ of $X$
- The total effect of $X$ on $Y$ is described in terms of comparisons of

$$E(Y_l|X_j) = \int E(Y|X_j, Z) p(Z|X_j) \, dZ$$

e.g. a mean difference $E(Y_l|X_2) - E(Y_l|X_1)$ or a log-OR

$$\log \left( \frac{E(Y_m|X_2)}{E(Y_l|X_2)} \right) - \log \left( \frac{E(Y_m|X_1)}{E(Y_l|X_1)} \right)$$
(Re)defining the effects for non-linear models

For a direct effect, define

\[ E_{(12)}^D(Y_l|X) = \int E(Y_l|X, Z) p_{(12)}(Z) dZ \]

where

\[ p_{(12)}(Z) = \frac{p(Z|X_1) + p(Z|X_2)}{2} \]

and compare

\[ E_{(12)}^D(Y_l|X_1) \text{ vs. } E_{(12)}^D(Y_l|X_2) \]
(Re)defining the effects for non-linear models

For an **indirect effect**, define

\[
E'_{(12)}(Y_i|X_j) = \int E_{(12)}(Y_i|Z) p(Z|X_j) \, dZ
\]

where

\[
E_{(12)}(Y_i|Z) = \frac{E(Y_i|X_1, Z) + E(Y_i|X_2, Z)}{2}
\]

and compare

\[
E'_{(12)}(Y_i|X_1) \text{ vs. } E'_{(12)}(Y_i|X_2)
\]
Decompositions of total effects

These quantities provide an exact partitioning of a total mean difference:

\[ E(Y_l|X_2) - E(Y_l|X_1) = [E_{(12)}^D(Y_l|X_2) - E_{(12)}^D(Y_l|X_1)] \\
+ [E_{(12)}^I(Y_l|X_2) - E_{(12)}^I(Y_l|X_1)] \]

For log odds ratios, corresponding additive decomposition is approximate but typically quite accurate.
Calculating the estimated effects

- First, need to specify models for $E(Y|X,Z)$ and $p(Z|X)$
  - Estimates of these are obtained in standard ways
- Second, the estimated effects are functions of estimates of $E(Y|X,Z)$ and $p(Z|X)$
  - For example, when intermediate variable $Z$ is discrete, this involves only summation, e.g.

$$
\hat{E}^D_{(12)}(Y_l|X_j) = \frac{1}{2} \sum_k \sum_{t=1,2} \hat{E}(Y_l|X_j, Z_k) \hat{p}(Z_k|X_t)
$$

- Third, standard errors of the estimated effects can be derived, ultimately from the standard errors of estimated parameters of $E(Y|X,Z)$ and $p(Z|X)$
Causal interpretations: Total effects

- Consider the counterfactual (potential outcomes) framework of formal causal inference
- Define potential outcomes (dropping subscript from $Y$):
  - $Y(x)$: value of $Y$ for a single subject when $X$ has value $x$
- **Total effect** of changing from $X = 1$ to $X = 2$ is defined in terms of comparisons of $Y(1)$ and $Y(2)$
- E.g. the mean difference (average treatment effect)

$$E\{Y(2)\} - E\{Y(1)\}$$

where expectation is over all subjects in a population
  - analogously for odds ratios etc.
Causal interpretations: Direct and indirect effects

- Define potential outcomes $Z(x)$ and $Y(x,z)$ similarly
  - Total effect can be expressed as
    \[
    E\{Y[2, Z(2)]\} - E\{Y[1, Z(1)]\}
    \]
  
- Natural direct effect of changing from $X = 1$ to $X = 2$ is
  \[
  NDE(1 \rightarrow 2) = E\{Y[2, Z(1)]\} - E\{Y[1, Z(1)]\}
  \]

and natural indirect effect is defined as either

\[
NIE(1 \rightarrow 2) = E\{Y[2, Z(2)]\} - E\{Y[2, Z(1)]\} \quad \text{or} \quad NIE(1 \rightarrow 2) = E\{Y[1, Z(2)]\} - E\{Y[1, Z(1)]\}
\]

e.g. Pearl (2001), Robins (2003), and [in a different framework] Geneletti (2007)
Causal interpretations: Direct and indirect effects

Estimates of the effects/associations defined in terms of $E(Y|X,Z)$ and $p(Z|X)$ above can be thought of as estimates of the following averages of natural effects:

- For direct effect:
  \[
  \frac{1}{2} [NDE(1 \rightarrow 2) + NDE(2 \rightarrow 1)]
  \]

- For indirect effect:
  \[
  \frac{1}{2} [NIE(1 \rightarrow 2) + NIE(2 \rightarrow 1)]
  \]

... at least under some fairly strict assumptions...
Conditions for causal interpretation

- Essentially, there should be no unmeasured confounders (common causes) of the relationships of $X$, $Z$ and $Y$
- Particularly problematic are confounders of the relationship of $Z$ and $Y$:

  ![Diagram showing causal relationships between $X$, $Z$, $W$, and $Y$]

- Such confounders should be controlled for in the estimation
Interpretation as associations: Total effects

- A more cautious interpretation than a causal one
  - ...and most that we can claim in the mobility example

- Consider first two groups:

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution of $X$</td>
<td>$X_1$ for all</td>
<td>$X_2$ for all</td>
</tr>
<tr>
<td>Distribution of $Z$</td>
<td>$p(Z</td>
<td>X_1)$</td>
</tr>
</tbody>
</table>

- i.e. observations with $X = X_1$ and with $X = X_2$, exactly as observed

- $\mathbb{E}(Y|X_1)$ and $\mathbb{E}(Y|X_2)$ are average expected values of $Y$ in these groups, when $\mathbb{E}(Y|X, Z)$ is as observed

- The total association is a comparison of these expected values
Interpretation as associations: Direct and indirect effects

- The **direct-effect association** is what would be observed when comparing average expected values of $Y$ between these two groups:

<table>
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<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ for all</td>
<td>$X_2$ for all</td>
<td></td>
</tr>
</tbody>
</table>

| Distribution of $Z$ | $[p(Z|X_1) + p(Z|X_2)]/2$ |

- i.e. groups which differ in $X$ but have the same distribution of $Z$

- **Indirect-effect association** analogously, comparing groups which differ in $p(Z|X_j)$ but have the same (even) mixture of $X_1$ and $X_2$ in both
Mobility example: Women in 1992

- Estimated (symmetric) log-odds ratios: total, direct and indirect

<table>
<thead>
<tr>
<th></th>
<th>I–S</th>
<th>W–I</th>
<th>W–S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed total effect</td>
<td>0.37</td>
<td>0.47</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Direct + Indirect effect</td>
<td>0.37</td>
<td>0.47</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Direct effect</td>
<td>0.07</td>
<td>0.25</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Indirect effect</td>
<td>0.30</td>
<td>0.22</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>% Indirect effect</td>
<td>80*</td>
<td>48</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>(18)</td>
<td>(9)</td>
<td>(7)</td>
</tr>
</tbody>
</table>

* Consistent with 100% indirect effect.
% of indirect (education) effect of total log-OR

Results in the example

% indirect effect

- I-S(W)
- W-I(W)
- W-S(W)
- I-S(M)
- W-I(M)
- W-S(M)
- I-S(W)
- W-I(W)
- W-S(M)
- I-S(M)

1973
1992

Path analysis for discrete variables

Methods Festival 2010
Future work

- Application to more recent British mobility data (1946, 1958 and 1970 birth cohort studies)
- Analysis with more detailed class classification
- Extensions to cases with more intervening variables
References


