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Using M-quantile models as an alternative to random effects to model the contextual value-added of schools in London

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Abstract. The measurement of school performance for secondary schools in England has developed from simple measures of marginal performance at age 16 to more complex contextual value-added measures that account for pupil prior attainment and background. These models have been developed within the multilevel modelling environment (pupils within schools) but in this paper we propose an alternative using a more robust approach based on M-quantile modelling of individual pupil efficiency. These efficiency measures condition on a pupils ability and background, as do the current contextual value-added models, but as they are measured at the pupil level a variety of performance measures can be readily produced at the school and higher (local authority) levels. Standard errors for the performance measures are provided via a bootstrap approach, which is validated using a model-based simulation.

JEL classification: C14, C21, I21.

Keywords: School Performance, Contextual Value-Added, M-Quantile Models, Pupil Efficiency, London.

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1. Introduction

The development of school performance measurement in England has seen a move from specialist studies by academic institutions (for example Tymms and Coe, 2003 review the work of the CEM Centre based at the University of Durham while Goldsten *et al*, 1993 is one example analysing a well-known dataset on Inner London schools) through to the initial use of simple school measures in national tables and the introduction of 'value-added' and the current 'contextual value-added' tables produced by DCFS (Ray, 2006). This current approach utilises multilevel modelling (Goldstein, 2003) to allow for the control of individual pupil characteristics such as prior attainment as well as more contextual factors such as the composition of pupil performance within the school, leaving the school 'random effect' (with a confidence interval) as a measure of the impact the school had on the pupil's performance that cannot be explained by measured characteristics of the pupil or their peers. These measures can be appropriate for judging school performance but as Leckie and Goldstein (2009) show, not school choice by parents, who need a measure of the school's performance several years into the future. In this paper we will concentrate on developing a measure of school performance appropriate for judging current performance and therefore school accountability but not appropriate for school choice. In addition, we will not impose apriori any structure on the data allowing the potential to compare performance of Local Authorities without the need to include explicit additional structure into our modelling. We apply the approach to evaluating schools in London using the linked National Pupil Database (NPD) / Pupil Level Annual School Census (PLASC) data set for the cohort of pupils in year 11 (age 16) in 2007/8. We produce appropriate measures of error to accompany our school performance measures as well as map the performance across Local Authorities showing that controlling for pupil characteristics goes some way to explaining differing performance at age 16 for pupils across London.

2. Approaches to Measuring School Performance

The recent paper by Leckie and Goldstein (2009) gives an excellent history of the development of school performance measurement in England resulting in the current contextual value-added models outlined in Ray (2006). In this section we review this approach and highlight some weaknesses. We propose an alternative approach that addresses some of the concerns expressed in relation to the current approach.

2.1. Reviewing the Current Approach

The current approach outlined in Ray (2006), and more flexible extensions by Leckie (2009) and Leckie and Goldstein (2009), essentially fits a linear model to the mean performance of pupils at age 16 corresponding to the end of compulsory schooling (referred to as key-stage 4), conditional on their prior attainment at age 11 (referred to as key-stage 2), the prior attainment of other pupils in the same cohort within their school, and other contextual factors such as whether the pupil receives free school meals (a means tested benefit associated with low income families) and the deprivation of the local area the pupil lives in. The pupil performance is measured by a score calculated from converting the grades in their eight best exams (typically GCSEs) to a points score. The exams are taken at around age 16 at the end of compulsory schooling in England. Often the performance measures are standardised, both at outcome and prior attainment, to be mean zero and standard deviation one to aid the interpretation of impacts but this is not necessarily required. The current CVA model (Ray, 2006) fits on the original scale of the variables while the recent work by Leckie and Goldstein (2009) transform the data onto a normal scale using the ranks of the original distributions. If the school has no additional impact on performance, coming say from the management structure within the school and its support of the teaching staff and pupil learning, the pupil residuals would be uncorrelated with each other. In reality, there is a residual school effect evidenced by a non-zero correlation across pupil residuals within schools (the mean of the pupil residuals is not zero) and therefore we can efficiently model the structure using a multilevel regression approach (Goldstein, 2003). The multilevel framework can then be extended considerably to allow for school effects over time (Leckie and Goldstein, 2009 is a recent paper covering this), cross-classified models to allow for the impact of local area (Leckie, 2009 going back to Garner and Raudenbush, 1991) as well as mobility (Leckie, 2009, and Goldstein, Burgess and McConnell, 2007).

The approach in the current models used by DCSF (Ray, 2006) applies the simple random effects specification giving a single measure of the school impact. The model assumes that this school impact is additive and constant across the pupils within a school. The impact comes from the school level residual in the random intercepts model and this is typically assumed to be normally distributed with independence between schools, independence of pupils within schools after controlling for the common school effect, and of course independence between the school residuals and the pupil level variables in the model. This final assumption results in the constant additive effect of the school. In reality, the assumptions of a normal distribution, constant effect, and even independence between schools can be problematic.

The problems partly come from the outcome being modelled. The score only takes the best eight GCSEs for each pupil to prevent a school entering pupils into lots of exams to inflate their overall score (Ray, 2006). However, the performance of the best performing pupils is essentially capped meaning that schools with higher performing pupils do not appear to add much value as a linear model can extrapolate that these pupils should do better than the capped score allows. Therefore the pupil residuals are lower and potentially forced to be negative leading to a low estimate of the school impact. In other words, the capping violates the constant variance assumptions of the model. We can deal with this within a multilevel framework by extending the level one error structure to capture non-constant variance (Goldstein, 2003) but this is an extension that has not been widely used. Goldstein and Thomas (1996) allow the pupil residual variance to vary by gender while Goldstein *et al* (1993) explored variation on pupil prior attainment. However, to the knowledge of the authors this approach has not been applied to the more recent nationally available pupil performance data and it is not the approach we take here to deal with this issue.

A related issue is the assumption of a uniform additive effect for the school across all pupils. This can be violated two ways. Firstly, as outlined in the previous paragraph, the non-constant variance and capping implies that adding the same absolute value to a pupil's performance does not have the same 'value' across the prior attainment range. It is easier to make absolute improvements at the bottom end of the scale. Secondly, even if this first issue is not a problem, the value added by the school may well depend on the pupil's characteristics. The second point can be incorporated by the use of random slopes at the school level (Goldstein and Thomas, 1996 is one example) and such adjustments are particularly relevant when parents are choosing a suitable school (Leckie and Goldstein, 2009). However, in this paper we are restricting to the use of school effects to judge current performance and therefore the single random intercept gives an average of the school's added value, even if it does not reflect the variable nature of that impact for the different pupils within the school.

2.2. An Alternative Framework for Pupil Performance and School Effects

In Section 2.1 we have highlighted some of the issues that occur with the current application of multilevel modelling. In this section we motivate an alternative approach that helps address some of these issues. We start by considering the individual performance of the pupils. Each pupil has a set of characteristics and context that drives their performance in the exams at age 16. If we compare across pupils with similar backgrounds we can start to consider their relative performance. This can be thought of as how efficient a pupil is with the particular set of circumstances they have and leads us to explore the literature in relation to production efficiency. More efficient pupils will perform better relative to those with similar inputs as measured by the prior attainment and similar 'production environment' measured by the contextual covariates. (Haveman and Wolfe, 1995 is an example from an Economic perspective viewing a child's achievement as the outcome of inputs by the pupil, their family and society or government.) Kokic *et al* (1997) introduce the use of m-quantile regression as a measurement of relative production-performance that they argue has good properties and we propose applying this technique to model the pupils' efficiency.

Once we have an efficiency measure for each pupil, we then impose the school structure. If the school has an impact it will allow pupils to be more efficient (inefficient) and so the average efficiency within a school will move away from the average efficiency across all pupils (around 0.5). This is similar to looking for correlation within schools in the pupil level errors. This aggregating of the individual quantile measures to get an 'area' summary links in with the recent extension of m-quantile models to small area estimation problems, where we wish to account for small area effects in our modelling (Chambers and Tzavidis, 2006). Interestingly, this school measure will then give a summary to aid judging the school performance but it will no longer necessarily satisfy the fourth criterion for measuring production-performance laid down by Kokic *et al* (1997). This is because the school impact for a group of pupils within the school can depend on the prior attainment and links to the concept of random slopes for schools in the multilevel literature.

This alternative approach still forces the idea of a constant school effect but this effect is no longer a simple additive impact across pupils. It actually represents an efficiency gain or loss relative to the population average of the pupils. The actual impact on a student's performance as measured by their exam scores will depend on the distribution of performance at their level of the covariates and this therefore allows for the differing variance in performance across the range of prior attainment. Related to this we also reduce the potential impact of capping as the approach recognises that at high levels of prior attainment the distribution of the outcome will be much tighter as a result of the capping but there will still be an ordering from the most to least efficient pupils.

As this approach is based on m-quantiles it is naturally more robust to distributional issues with the outcome variable than a standard multilevel approach. An alternative approach taken by Leckie and Goldstein (2009) to address this issue is via a transformation of the data to make it better approximate a normal distribution. We prefer to try a modelling approach that is robust rather than a transformation approach. In addition, the transformation approach does not tackle the issue of the non-constant residual variance and this requires more complex random structures, such as a random slope at level one on prior attainment, increasing further the complexity of the model.

An additional advantage of this approach is that we get the full distribution of the pupil efficiencies within schools but this is driven by the data rather than a distributional assumption imposed apriori on the pupil level residuals. Therefore we can summarize the school effect as not only the mean but other summaries such as the median or the proportion of pupils within a school above the upper quartile. Also, as we have not imposed structure on the model so we can summarize pupil performance at the local authority level (the local administrative units within London) or compare the performance of groups within school. In the subsequent analysis we will demonstrate these aspects by mapping performance at the local authority level across London and comparing the performance within mixed schools for males and females. Of course, the multilevel framework can also provide a measure at the local authority level by extending to three levels and a school level random slope on gender (Goldstein and Thomas, 1996) would allow for a difference in the school impact by gender. However, we see the advantage of our approach being you model the pupil level performance and can then explore performance at different levels (schools, local geography, sub-groups) without having to pre-specify them in the model.

2.3. Outline of the Paper

In Section 3 of the paper we introduce the reader to m-quantile modelling with the necessary technical detail and formally state our measure of performance at the school level. In Section 4 we then outline an application of a non-parametric bootstrap to estimate the standard error of any efficiency measure, which is crucial if we are to use the performance measure (Goldstein and Speigelhalter, 1996) to make judgements about the relative performance of institutions or other sub-groups. This is supported by a small model-based simulation. In Section 5 we apply the approach to data for pupils and schools in London in 2008 and in Section 6 we present illustrative results at the school and local authority level. Finally in Section 7 we draw some conclusions from the analysis.

3. Modelling Conditional Quantiles

The classical theory of linear statistical models is a theory of conditional expectations. That is, a regression model summarises the behaviour of the mean of Y at each point in a set of $\boldsymbol{X}$'s (Mosteller and Tukey, 1977). Unfortunately, this summary provides a rather incomplete picture, in much the same way as the mean gives an incomplete picture of a distribution. It is usually much better to fit a family of regression models, each one summarising the behaviour of a different percentage point (quantile) of Y at each point in this set of $\boldsymbol{X}$'s. This can be achieved using quantile regression. This demonstrated in Figure 1, which plots in blue pupil performance at 16 against their prior attainment at 11. The red dots show the fitted mean for a quadratic regression of pupil performance at 16 using performance at 11 as a prior attainment, while the grey dots show quantiles for the same model fitted at 1%, 5%, 25%, 50%, 75%, 95% and 99%.

[Figure 1 Here]

The seminal paper by Koenker and Bassett (1978) is usually regarded as the first detailed development of quantile regression. In the linear case, quantile regression leads to a family of planes indexed by the value of the corresponding percentile coefficient $q \in (0,1)$. For each value of q, the corresponding model shows how the q^{th} quantile of the conditional distribution of Y given $\boldsymbol{X}$, varies with $\boldsymbol{X}$. For example, when $q = 0.5$ the quantile regression line shows how the median of this conditional distribution changes with $\boldsymbol{X}$. Similarly, when $q = 0.1$ this regression line separates the top 90% of the conditional distribution from the lower 10%. A linear model for the q^{th} conditional quantile of Y given a vector of covariates **X** is

$$Q_q(Y \mid \boldsymbol{X}) = \boldsymbol{X}^T \boldsymbol{\beta}_q \ , \tag{1}$$

and $\boldsymbol{\beta}_q$ is estimated by minimising $\sum_{i=1}^{n} \left| y_i - \mathbf{x}_i^T \boldsymbol{\beta} \right| \left\{ (1-q) I\left(y_i - \mathbf{x}_i^T \boldsymbol{\beta} \le 0 \right) + qI\left(y_i - \mathbf{x}_i^T \boldsymbol{\beta} > 0 \right) \right\}$ with respect to $\boldsymbol{\beta}$. Solutions to this minimisation problem are usually obtained using linear programming methods (Koenker and D'Orey, 1987). Functions for fitting quantile regression now exist in standard statistical software, e.g. the. R statistical package (R Development Core Team, 2004).

3.1. Using the *M*-quantile approach

Quantile regression can be viewed as a generalisation of median regression. In the same way, expectile regression (Newey and Powell, 1987) is a "quantile-like" generalisation of mean regression. *M*-quantile regression (Breckling and Chambers, 1988) integrates these concepts within a common framework defined by a "quantile-like" generalisation of regression based on influence functions (M-regression).

The *M*-quantile of order q for the conditional density of Y given $\boldsymbol{X}$ is defined as the solution $Q_q(\boldsymbol{X};\psi)$ of the estimating equation $\int \psi_q (Y - Q) f(Y \mid \boldsymbol{X}) dY = 0$, where ψ denotes the influence function associated with the *M*-quantile. A linear *M*-quantile regression model is one where we assume that

$$Q_q(Y \mid \boldsymbol{X};\psi) = \boldsymbol{X}^T \boldsymbol{\beta}_\psi (q), \tag{2}$$

that is, we allow a different set of regression parameters for each value of *q*. For specified *q* and ψ, estimates of these regression parameters can be obtained by solving the estimating equations

$$\sum_{i=1}^{n} \psi_q (r_{iq\psi}) \boldsymbol{X}_i = \mathbf{0}, \tag{3}$$

where $r_{iq\psi} = Y_i - \boldsymbol{X}_i^T \boldsymbol{\beta}_\psi (q)$, $\psi_q (r_{iq\psi}) = 2\psi(s^{-1} r_{iq\psi}) \left\{ qI(r_{iq\psi} > 0) + (1-q) I(r_{iq\psi} \le 0) \right\}$ and s is a suitable robust estimate of scale, e.g. the MAD estimate $s = median \left| r_{iq\psi} \right| / 0.6745$. In this paper we use a Huber Proposal 2 influence function, $\psi(u) = uI(-c \le u \le c) + c\,\mathrm{sgn}(u)$. Provided c is bounded away from zero, estimates of $\boldsymbol{\beta}_{\psi}(q)$ are obtained by iterative weighted least squares. The steps of the algorithm are as follows:

1. Start with initial estimates $\boldsymbol{\beta}_{\psi}(q)$ and *s*;
2. Form residuals $r_{iq\psi}$
3. Define weights $w_i = \psi_q\left(r_{iq\psi}\right)/r_{iq\psi}$
4. Update $\boldsymbol{\beta}_{\psi}(q)$ using weighted least squares regression with weights w_{i};
5. Iterate until convergence.

These steps can be implemented in R by a straightforward modification of the IWLS algorithm used for fitting M-regression (Venables and Ripley, 2002, section 8.3).

M-quantile regression is synonymous to outlier robust estimation. However, an advantage of *M*-quantile regression is that it allows for more flexibility in modelling. For example, the tuning constant *c* can be used to trade robustness for efficiency in the *M*-quantile regression fit, with increasing robustness/decreasing efficiency as we move towards quantile regression and decreasing robustness/increasing efficiency as we move towards expectile regression.

3.2. An *M*-quantile measure of school performance

Let us assume that the output of a school *j* can be measured by a single variable *Y* for example, GCSE performance and that this output is associated with a set of explanatory (input) variables $\boldsymbol{X}$ and for the time being, let us assume that we have data only at school level. Kokic et al. (1997) proposed a measure of production performance that is based on the use of *M*-quantile models. Let us assume that the quantiles of the conditional distribution $f\left(Y \mid \boldsymbol{X}\right)$ can be model using a linear function as in (2).

Using (2), the *M*-quantile measure of performance is defined as follows: If the *q*th *M*-quantile surface implied by (2) passes through $\left(Y_{\mathrm{j}}, \boldsymbol{X}_{\mathrm{j}}\right)$, the performance measure for the *j*th school is $p_{\mathrm{j}} = q$ and the higher the value of p_{j} the better the school performance. Until this point we have assumed that data are available only at school level. In most cases, however, data are available for pupils clustered with schools. This creates a two level hierarchical structure, which we should account for. Below we use Y_{ij} and $\boldsymbol{X}_{ij}$ to denote the data for pupil *i* in school *j*.

Multilevel models assume that variability associated with the conditional distribution of *Y* given $\boldsymbol{X}$ can be at least partially explained by a pre-specified hierarchical structure. The idea of measuring hierarchical effects via an *M*-quantile model has recently attracted a lot of interest in the small area estimation literature (Chambers and Tzavidis 2006; Tzavidis et al. 2010) and also in other applications (Yu and Vinciotti). Following the development in Chambers and Tzavidis (2006), we characterise the conditional variability across the population of interest by the *M*-quantile coefficients of the population units. For unit *i* in cluster (school) *j* with values Y_{ij} and $\boldsymbol{X}_{ij}$, this coefficient is the value p_{ij} such that $Q_{p_{ij}}(\boldsymbol{X}_{ij};\psi)=Y_{ij}$. Note that these *M*-quantile coefficients are determined at the population level. If a hierarchical structure does explain part of the variability in the population data, we expect units within the clusters defined by this hierarchy to have similar *M*-quantile coefficients. Consequently, we characterise a cluster by the location of the distribution of its associated unit (pupil)-level *M*-quantile coefficients. In particular, the *M*-quantile measure of performance is defined as

$$p_j = N_j^{-1}\sum_{i=1}^{N_j} q_{ij}\,. \tag{4}$$

The measure of performance defined by (4) is an extension of the *M*-quantile measure of performance proposed by Kokic et al. (1997) that accounts for the hierarchical structure of the data. Estimation of (4) is performed as follows. Following Chambers and Tzavidis (2006), we first estimate the M-quantile coefficients $\{q_i; i\in s\}$ of the sampled units without reference to the groups (schools) of interest. A grid-based procedure for doing this under (3) is described by Chambers and Tzavidis (2006) and can be used directly with (4). We first define a fine grid of *q* values in the interval (0,1). Chambers and Tzavidis (2006) use a grid that ranges between (0.01 to 0.99) with step 0.01. We employ the same grid definition and then use the sample data to fit (3) for each distinct value of *q* on this grid. The M-quantile coefficient for unit *i* with values Y_i and $\boldsymbol{X}_i$ is calculated by using linear interpolation over this grid to find the unique value q_i such that $\hat{Q}_{q_i}(\boldsymbol{X}_i;\psi)\approx Y_i$. A school j specific M-quantile measure of performance, $\hat{p}_j$ is then estimated by the average value of the unit (pupil) M-quantile coefficients in school j,

$$\hat{p}_j = n_j^{-1}\sum_{i=1}^{n_j}\hat{q}_{ij}\,. \tag{5}$$

The advantage of this approach is that we have the efficiencies, q_i, for each pupil *i* and therefore the averaging done in (5) to give the school value can be done also across a higher level of geography (such as local authority) or done across pupil sub-groups such as males and females within schools without the need to pre-specify complex structures in the modelling.

4. Mean Squared Error Estimation

In this section we describe a non-parametric bootstrap approach to MSE estimation of the *M*-quantile measure of school performance that is based on the approach of Lombardia et al. (2003) and Tzavidis et al. (2010). In particular, we define two bootstrap schemes that resample residuals from an *M*-quantile model fit. The first scheme draws samples from the empirical distribution of suitably re-centred residuals. The second scheme draws samples from a smoothed version of this empirical distribution. Using these two schemes, we generate a bootstrap population, from which we then draw bootstrap samples.

4.1. Implementing the Bootstrap

In order to define the bootstrap population, we first calculate the *M*-quantile model residuals $e_{ij} = y_{ij} - \mathbf{x}_{ij}^T\hat{\boldsymbol{\beta}}_\psi(q)$. A bootstrap finite population $U^* = \{y_{ij}^*, \mathbf{x}_{ij}\}, i \in U, j = 1,\ldots,d$ with $y_{ij}^* = \mathbf{x}_{ij}^T\hat{\boldsymbol{\beta}}_\psi(q) + e_{ij}^*$ is then generated, where the bootstrap residuals e_{ij}^* are obtained by sampling from an estimator of the CDF $\hat{G}(u)$ of the e_{ij}. In order to define $\hat{G}(u)$, we consider two approaches: (i) sampling from the empirical CDF of the residuals e_{ij} and (ii) sampling from a smoothed CDF of these residuals. In each case, sampling of the residuals can be done in two ways: (i) by sampling from the distribution of all residuals without conditioning on the group (the unconditional approach); and (ii) by sampling from the conditional distribution of residuals within the group *j* (the conditional approach). The empirical CDF of the residuals is

$$\hat{G}(u) = n^{-1}\sum_{j=1}^{d}\sum_{i\in s_j} I(e_{ij} - \overline{e}_s \le u)\,, \tag{6}$$

where $\overline{e}_s$ is the sample mean of the e_{ij}. Similarly, the empirical CDF of these residuals in group *j* is

$$\hat{G}_j(u) = n_j^{-1} \sum_{i \in s_j} I(e_{ij} - \overline{e}_{sj} \le u), \tag{7}$$

where $\overline{e}_{sj}$ is the sample mean of the e_{ij} in group *j*. A smoothed estimator of the unconditional CDF is

$$\hat{G}(u) = n^{-1} \sum_{j=1}^{d} \sum_{i \in s_j} K\left\{h^{-1}\left(u - e_{ij} + \overline{e}_s\right)\right\}, \tag{8}$$

where h > 0 is a smoothing parameter and *K* is the CDF corresponding to a bounded symmetric kernel density *k*. Similarly a smoothed estimator of the conditional CDF in group *j* is

$$\hat{G}_j(u) = n_j^{-1} \sum_{i \in s_j} K\left\{h_j^{-1}\left(u - e_{ij} + \overline{e}_{sj}\right)\right\}, \tag{9}$$

where $h_j > 0$ and *K* are the same as above. In the empirical studies reported in Section 4.2, we define *K* in terms of the Epanechnikov kernel, $k(u) = (3/4)\left(1 - u^2\right) I\left(|u| < 1\right)$, while the smoothing parameters h and h_j are chosen so that they minimize the cross-validation criterion suggested by Bowman et al. (1998). That is, in the unconditional case, h is chosen in order to minimize

$$CV(h) = n^{-1} \sum_{j=1}^{d} \sum_{i \in s_j} \int \left[I\left\{(e_{ij} - \overline{e}_s) \le u\right\} - G_{-i}(u) \right]^2 du, \tag{10}$$

where $G_{-i}(u)$ is the version of $G(u)$ that omits sample unit *i*, with the extension to the conditional case being obvious. It can be shown (Li and Racine, 2007, section 1.5) that choosing h and h_j in this way is asymptotically equivalent to using the MSE optimal values of these parameters.

In what follows we denote by p_j the unknown true *M*-quantile measure of school performance, by $\hat{p}_j$ the estimator of p_j based on sample s_j, by p_j^* the known true *M*-quantile measure of school performance of the bootstrap population U_j^*, and by $\hat{p}_j^*$ the estimator of p_j^* based on bootstrap sample s_j^*.

We estimate the MSE of the *M*-quantile measure of school performance as follows. Starting from the sample *s,* we generate *B* bootstrap populations, U^{*b}, using one of the four abovementioned methods for estimating the CDF of the residuals. From each bootstrap population, U^{*b}, we select L samples using simple random sampling without replacement within the schools with $n_j^* = n_j$. The bootstrap estimator of the MSE of $\hat{p}_j$ is then

$$\hat{MSE}_j = B^{-1}L^{-1}\sum_{b=1}^{B}\sum_{l=1}^{L}\left\{\hat{p}_j^{*bl} - \text{av}_L(\hat{p}_j^{*bl})\right\}^2 + \left\{B^{-1}L^{-1}\sum_{b=1}^{B}\sum_{l=1}^{L}\left(\hat{p}_j^{*bl} - p_j^{*b}\right)\right\}^2 . \qquad (11)$$

In (12) p_j^{*b} is the school *j* value of the characteristic of interest for the *b*th bootstrap population and $\text{av}_L(\hat{p}_j^{*bl}) = L^{-1}\sum_{l=1}^{L}\hat{p}_j^{*bl}$, where $\hat{p}_j^{*bl}$ is the estimator of this characteristic computed from the *l*th sample of the *b*th bootstrap population, (*b* = 1,…,*B*, *l* = 1,…,*L*). Note that this bootstrap procedure can also be used to construct confidence intervals for the value of p_j by 'reading off' appropriate quantiles of the bootstrap distribution of $\hat{p}_j$.

4.2. Monte-Carlo evaluation of the MSE estimator

A small scale model-based Monte-Carlo simulation study was designed for evaluating the performance of the bootstrap MSE estimator of the *M*-quantile measure of performance. The population data on which this simulation was based is generated from a 2-level random intercepts model (individuals nested within groups (schools)). The model parameters used for generating the Monte-Carlo populations are obtained from fitting a 2-level random intercepts model to the 2008 linked NPD / PLASC data. The outcome variable is the post attainment, we control for the effect of prior attainment and the random effects are specified at the school level. In particular, population data are generated using $y_{ij} = -57 + 14x_{ij} + \gamma_j + \varepsilon_{ij}$, where $x_{ij} \sim N(27,4)$, $\gamma_j \sim N(0,17.15)$ and $\varepsilon_{ij} \sim N(0,65.13)$. We generate in total 250 populations with a total size 16020 units in 40 schools. The school population sizes range from 200 to 590 with an average of 400 units per school. From each of the 250 populations we take independent samples by randomly selecting pupils within the 40 schools. The group sample sizes range from 20 to 59 with an average of 40 units per group. For each sample, estimates of the *M*-quantile measure of performance were obtained using the methodology of Section 3 and an *M*-quantile model that included as main effect x_{ij}. For each Monte-Carlo simulation bootstrap MSE estimation for the *M*-quantile measure of performance was implemented by generating a single bootstrap population and then taking *L* = 250 bootstrap samples from this population. The bootstrap population was generated unconditionally, with bootstrap population values obtained by sampling from the smoothed residual distribution generated by the sample data obtained in each Monte Carlo simulation. The performance of the MSE estimator is assessed using the percentage relative bias of the bootstrap MSE estimator defined by

$$RB(M) = \underset{i}{mean}\left\{ M_i^{-1} K^{-1} \sum_{k=1}^{K} \left(\hat{M}_{ik} - M_i \right) \right\} \times 100 . \tag{12}$$

Here the subscript *i* indexes the schools and the subscript *k* indexes the *K* Monte Carlo simulations, with $\hat{M}_{ik}$ denoting the simulation *k* value of the MSE estimator in school *i*, and M_i denotes the actual (i.e. Monte Carlo) MSE in area *i*. In addition to the relative bias we compute coverage rates of 95% confidence intervals which are constructed using the M-quantile measure of school performance plus or minus twice its estimated standard error. The coverage rate is defined as the number of times this confidence interval includes the true M-quantile measure of performance and for a 95% confidence interval this rate must be close to 95%.

The results from this simulation studies are set out in Table 1 and Figure 2. The bootstrap MSE estimator tracked the true (empirical MSE over Monte-Carlo simulations) MSE of the M-quantile measure of school performance and provided coverage rates that were close to the nominal 95%. On average, the relative bias was very low (1%-2%) and for the majority of the schools this bias did not exceed 5%. The maximum relative bias was 14%, however, relative percentage figures must be interpreted with care in this case as the MSE estimates are small values. This is apparent by looking at the numbers of the actual and estimated MSEs for the school with the highest relative bias. This is school 34 and the estimated MSE is 0.0366 whereas the actual (Monte-Carlo) MSE is 0.032. These results suggest that the non-parametric bootstrap scheme we proposed in Section 4 can be reliably used for estimating the MSE of the M-quantile measure of school performance.

[Table 1 Here]

[Figure 2 Here]

5. Modelling Pupil Performance across London

In this section we now turn to the full application of the approach to pupils in schools in London. We utilise the linked NPD/PLASC data to provide information on pupils background, their performance at age 11 (prior attainment) and their performance at age 16 (the outcome). To make the demonstration more straightforward we start with the 81,882 pupils that are sitting exams within the right cohort (age 16 during the 2007/08 academic year). We then select those that have performance information at both time-points (we lose just under 10,000 pupils), linked PLASC data to provide the background information (we lose just under 5,900 pupils), and finally we drop three schools that each contain a single pupil. This leaves us with 66,209 pupils.

5.1. Modelling the Pupil Performance

For the purposes of showing what can be achieved with this approach, we utilise a model specification similar to the CVA model outlined in Ray (2006) and readers should refer to this for a detailed motivation of the model specification. The outcome (pupil performance at 16) is a score based in the pupils eight best exams. These are typically GCSEs (the standard exam taken at age 16 at the completion of compulsory education in England) and eight GCSEs with top grades of A* give a total score of 464. Pupils can exceed this maximum by taking exams in a few subjects at a more advanced level at age 16 but for most pupils this creates a cap to their performance measure. Prior attainment at age 11 is captured by the pupils mean performance across Mathematics, English and Science (which has a quadratic relationship through the inclusion of a squared term) as well as the differences between the pupil's mean and their individual scores in Mathematics and English. We also control for the school level mean in prior attainment and the standard deviation of pupil prior attainment within school. (For simplicity, we use the pupils' mean prior attainment at age 11 to calculate these school level variables while Ray, 2006 uses a measure at age 14.) To control for pupil background we include an indicator for receipt of free school-meals as a proxy for the pupil coming from a low income family, an indicator of local area income for the pupils' home address, as well as indicators of the pupils' age within the school year, gender, ethnicity, first language at home, special educational needs, and movement across local authority boundaries between ages 11 and 16.

5.2. Brief Discussion of the fitted Models

The motivation behind this paper is not to define a revised model specification at the pupil level for contextual value-added modelling of school performance, but rather propose an alternative framework in which to explore the schools' performance. However, to demonstrate that the *M*-quantile modelling is behaving as would be expected, Table 2 compares the model parameters from the standard CVA approach (two-level random intercepts model with pupils within schools) against the *M*-quantile median line.

[Table 2 Here]

From Table 2 we can see the same pattern emerging for both models, such as the lower performance (conditional on all other factors) for those pupils receiving free school meals. Some of the effects, while in the same direction are less pronounced in the median model such as the quadratic shape for the average prior attainment, which may indicate the presence of some outliers in the data. Standard errors are provided with both models. We should note that while the random effects model has adjusted its standard errors for correlation within schools the M-quantile model has not (and so its standard errors are likely under-estimated). This is not an issue for the performance measure as the grouping structure is taken into after estimating the individual efficiencies and the bootstrap approach we use (outlined in Section 4) does account for the appropriate structure. However, it does raise the importance issue of model fitting and the specification of the model structure, which we have not dealt with here given we are reproducing the standard CVA model.

6. Evaluating the School and Local Authority Impact

In this section, we now use the results from the pupil level modelling in Section 5, which results in the q_i's for each individual pupil being estimated, to construct a measure of school performance (see Section 3.2) as well as exploring within schools and at a higher level of aggregation comparing across the local authorities in London.

6.1. School Performance

We define the school performance as the mean of the pupils' q_i's and as this moves away from the overall pupil mean of 0.52 it represents the school adding to the efficiency of the pupils or reducing the efficiency of the pupils. Of course, to judge this we need a confidence interval around the school efficiency measure and this can be calculated via an estimated standard error resulting from the bootstrap (see Section 4). These confidence intervals can be adjusted to allow for multiple pair-wise comparisons between schools (Goldstein and Healy, 1995) but here we just present estimates with standard 95% confidence intervals to illustrate the approach. Figure 3 presents this information as a caterpillar plot for six mixed schools from across London with a range of estimated school efficiencies. The schools were chosen to have relatively small errors on the overall q's to demonstrate the potential impacts that can be seen across schools.

[Figure 3 Here]

As is common in these situations (see for example Leckie and Goldstein, 2009), the width of these 95% confidence intervals demonstrate how careful we should be regarding comparing schools to the overall mean or comparisons between schools (for which adjusted confidence intervals would be needed). In addition, as these are mixed schools the efficiency of the school has been calculated separately for boys and girls with corresponding standard errors and confidence intervals. As the sample sizes for these gender specific efficiencies are smaller the confidence intervals of course become correspondingly wider. This makes finding truly significant differences difficult but interestingly in five of the six schools the school efficiency for girls is higher, and this is after we have controlled for gender at the individual pupil level.

6.2. Comparing Performance across Local Authorities

Our modelling approach at the pupil level does not apriori impose a structure on the data and this then allows us to produce an average performance measure at the local authority level (averaging the pupils' q_i's across local authorities rather than individual schools). However, the bootstrap outlined in Section 4 samples pupil residuals within schools when applied at the school level and so to produce standard errors at the local authority level we re-run the bootstrap respecting this structure rather than the school structure.

We compare across the local authorities by mapping the average q's. Figure 4a is the marginal performance of each local authority based on averaging the pupils' q_i's from a null (intercept only) model so that we are comparing like with like. When making these comparisons and interpreting the measures it is important to remember the *M*-quantiles do not exactly match the empirical distribution so the empirical mean and median are not usually 0.5. The four colours represent a group of local authorities clearly below the marginal mean for the pupils, one spanning the mean (around 0.53), and two above the mean showing the positively skewed nature the performance measures. Figure 4b is the conditional or 'contextual value-added' performance mapped using the same legend to allow easy visual comparison.

[Figure 4 Here]

Comparing the two maps clearly shows less variation across the local authorities, once pupil background and context has been controlled. In particular, the highest performing group on the marginal map does not exist on the conditional map. In addition, the large area of poor performing local authorities east of central London on the marginal map have all generally improved once the controls are introduced. However, these conditional performance measures are still subject to variability and therefore Figure 5 uses three colours to highlight those local authorities with performance significantly above the overall mean of the conditional q's (0.52), those significantly below, and those in the middle based on whether the estimated 95% confidence interval around the local authority estimates includes the overall mean.

[Figure 5 Here]

Figure 5 reveals two areas in south London that can be considered significantly below the mean and across London there are some areas significantly above, but many cannot be considered different from the mean. This again warns against simply ranking based on the q's due to the uncertainty.

7. Discussion

In this paper we have introduced an alternative framework for assessing the impact of schools on their pupils' performance. The standard approach to contextual value-added modelling uses a random effects model to account for the residual impact of the school. However, this approach makes fairly strong distributional assumptions and leads to a single measure. More complex comparisons require more complex structures within the models. Our alternative approach utilises the robustness of M-quantiles and as it leads to a measure of efficiency for the individual pupils we can summarize performance at a variety of levels without requiring additional structure in the modelling. We have chosen to use the mean, but as we have modelled the entire distribution of performance, we can produce other summaries such as the proportion of pupils within schools coming in the top 25% of the overall distribution to highlight schools that particularly contribute to top pupil's having high efficiency.

To explore a differing impact of the school by gender within the multilevel framework would require a random slope at the school level on gender. Unlike our approach, this would give an overall impact of gender differences at the school level but to do this imposes a structure in relation to the variance of the school level random effects by gender. The approach we have used here does not impose any overall structure (we cannot say there is a general impact of gender on school effects) but is therefore flexible in terms of the actual impacts at the level of the individual school. However, with both approaches finding significant impacts within individual schools will likely be difficult due to the uncertainty across pupils at this level, as shown in Figure 3. Exploring higher level impacts, such as the local authorities we have looked mapped (Figures 4 and 5), can again be achieved within the multilevel framework by extending the fitted model to include the extra level. However, as our approach has estimated the efficiency at the pupil level we can explore the impact of this level without the need for additional modelling.

One issue we should acknowledge is that of model fitting. In this work we have utilised a similar model specification to the standard CVA model (Ray, 2006) and so standard errors on the model parameters were not needed to judge the model specification. However, standard errors are produced with each model (Table 2 presents them for the median) but, as noted in section 5, these will not be adjusted to account for any higher level clustering in the data. The issue of appropriate standard errors for model parameters is therefore a future area of research, although the standard errors on the performance measures calculated via the bootstrap do adjust for clustering in the population so that Figure 3 and 5 do give a fair picture of quality, given a model specification.

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Table 1

Across schools distribution of the true (i.e. Monte Carlo) mean squared error, average over Monte Carlo simulations of estimated mean squared error, relative bias (%) of the bootstrap MSE estimator and coverage rates of nominal 95% confidence intervals for the M-quantile measure of school performance (5).

MSE	Percentiles of across schools distribution					
	Min	25th	50th	Mean	75th	Max
True	0.027	0.033	0.040	0.041	0.044	0.067
Bootstrap	0.028	0.034	0.040	0.041	0.043	0.065
Relative Bias (%)	-7.27	-2.21	0.98	2.01	5.04	14.51
Coverage	0.928	0.940	0.948	0.952	0.961	0.984

The estimated mean squared error is based on (11) using the smoothed unconditional approach. Intervals were defined as the M-quantile measure of school performance estimated by (6) plus or minus twice its estimated standard error, calculated as the square root of (11).

Table 2

Comparison of the coefficients from the standard random effects approach to CVA with the coefficients from the median M-quantile model

Variable	CVA Model		M-quantile Median Model	
	Estimate	Standard Error	Estimate	Standard Error
Const	151.52	24.80	110.08	8.76
KS2. Average Score (fine grade)	-4.19	0.55	-1.42	0.45
KS2 Average Score (squared)	0.31	0.01	0.26	0.01
KS2 Difference from Average (English)	0.36	0.08	0.40	0.06
KS2 Difference from Average (Mathematics)	0.23	0.09	0.22	0.07
Pupil receives Free School Meals	-7.55	0.64	-5.08	0.53
IDACI Score (Pupil's Home Address)	-30.09	1.56	-24.29	1.14
Pupil is Male	-13.13	0.57	-13.67	0.40
Age within School Year	-0.80	0.07	-0.69	0.06
English As First Language	-16.47	0.82	-13.83	0.65
Not SEN	Ref		Ref	
sen1	-39.43	1.63	-33.36	1.35
sen2	-35.18	0.66	-26.26	0.52
White British	Ref		Ref	
White Other	14.57	1.06	15.93	0.85
Black African	23.88	1.02	21.34	0.82
Black Caribbean	9.45	1.02	7.86	0.81
Black Other	11.53	1.71	9.60	1.40
Indian	22.31	1.27	21.88	0.97
Pakistani	18.72	1.52	17.39	1.20
Bangladeshi	21.47	1.65	18.69	1.21
Chinese	32.18	2.75	28.67	2.28
Asian Other	28.43	1.58	27.26	1.29
Mixed	5.59	1.00	5.77	0.82
Other	21.69	1.43	23.00	1.16
Unknown	4.66	4.15	9.73	3.45
Different LA (KS2 to KS4)	-2.21	0.62	-0.27	0.48
School Mean (KS2 Average Score)	3.41	0.68	3.48	0.18
School Standard Deviation (KS2 Average Score)	-1.67	1.59	-1.00	0.38

Figure 1: Comparing the fitted mean line (red) with fitted lines (grey) covering quantiles at 1%, 5%, 25%, 50%, 75%, 95% and 99%.

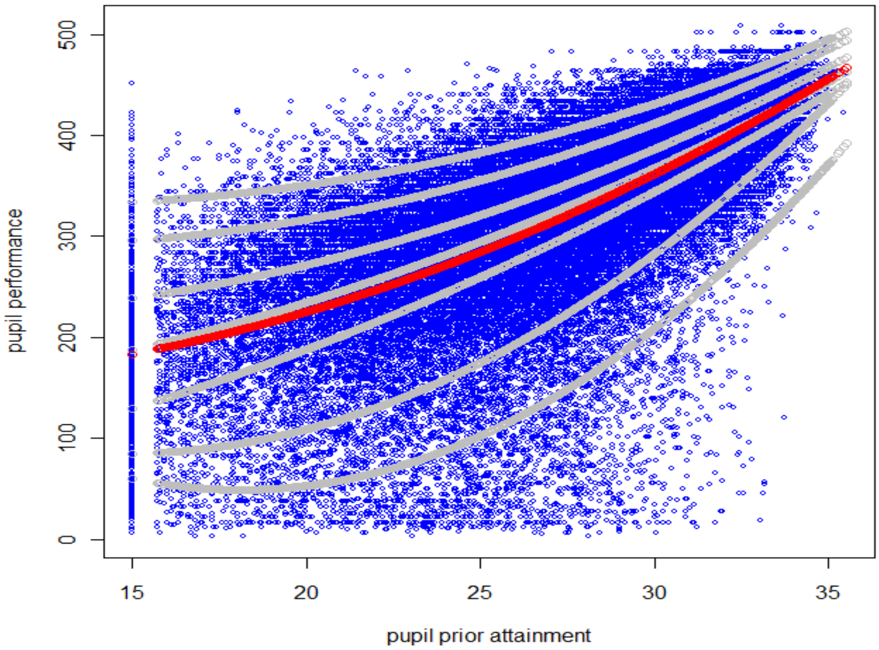


Figure 2: True (empirical) Mean Square Error (Red Line) and estimated Mean Square Error (black line) of the M-quantile measure of school performance. The Estimated mean squared error based on (11) using the smoothed unconditional approach.

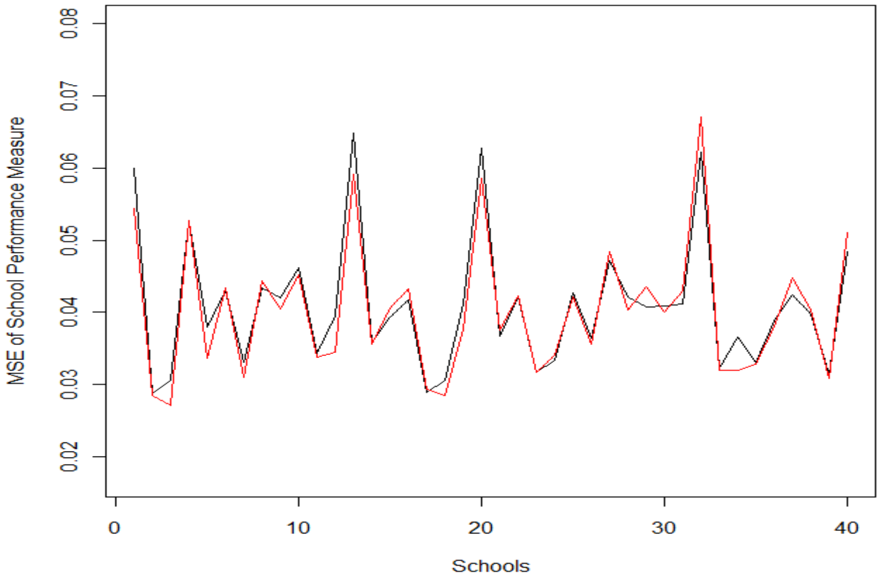


Figure 3: Caterpillar plot for school comparing males and females

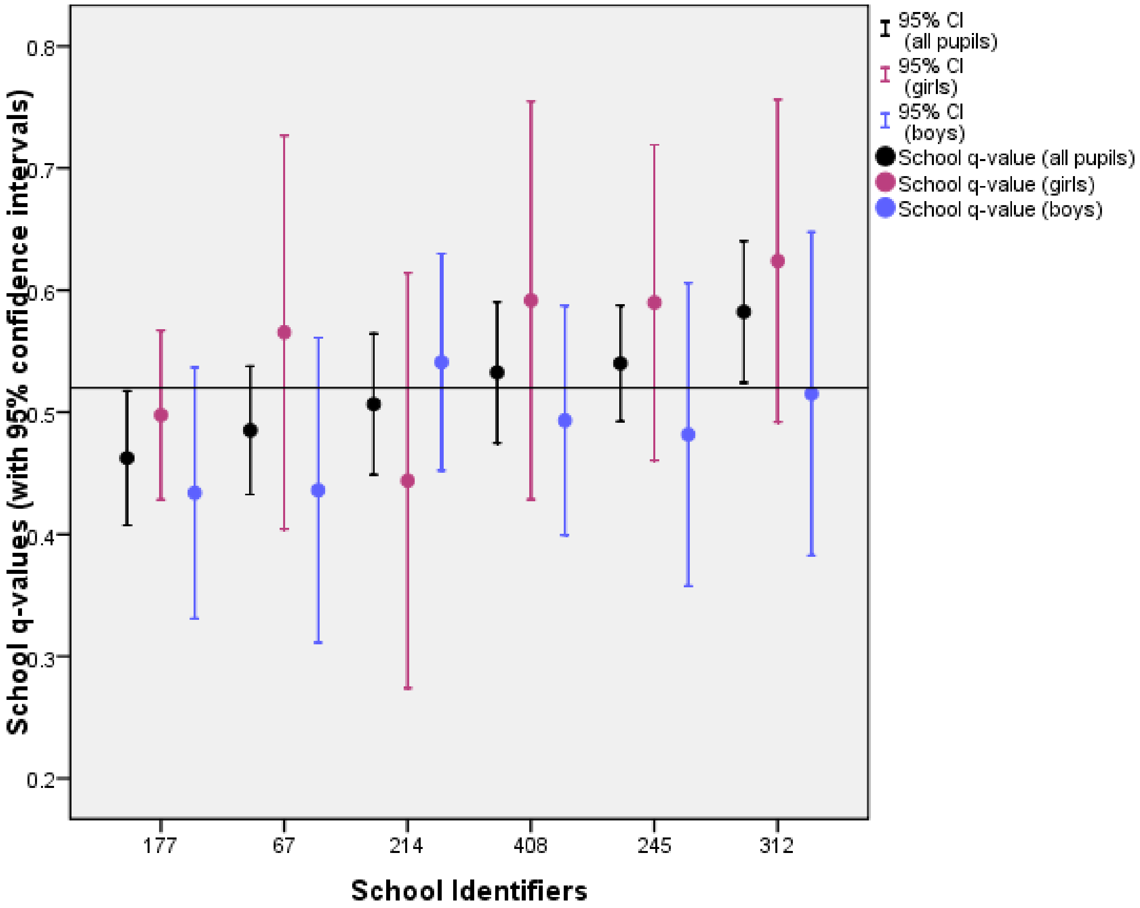
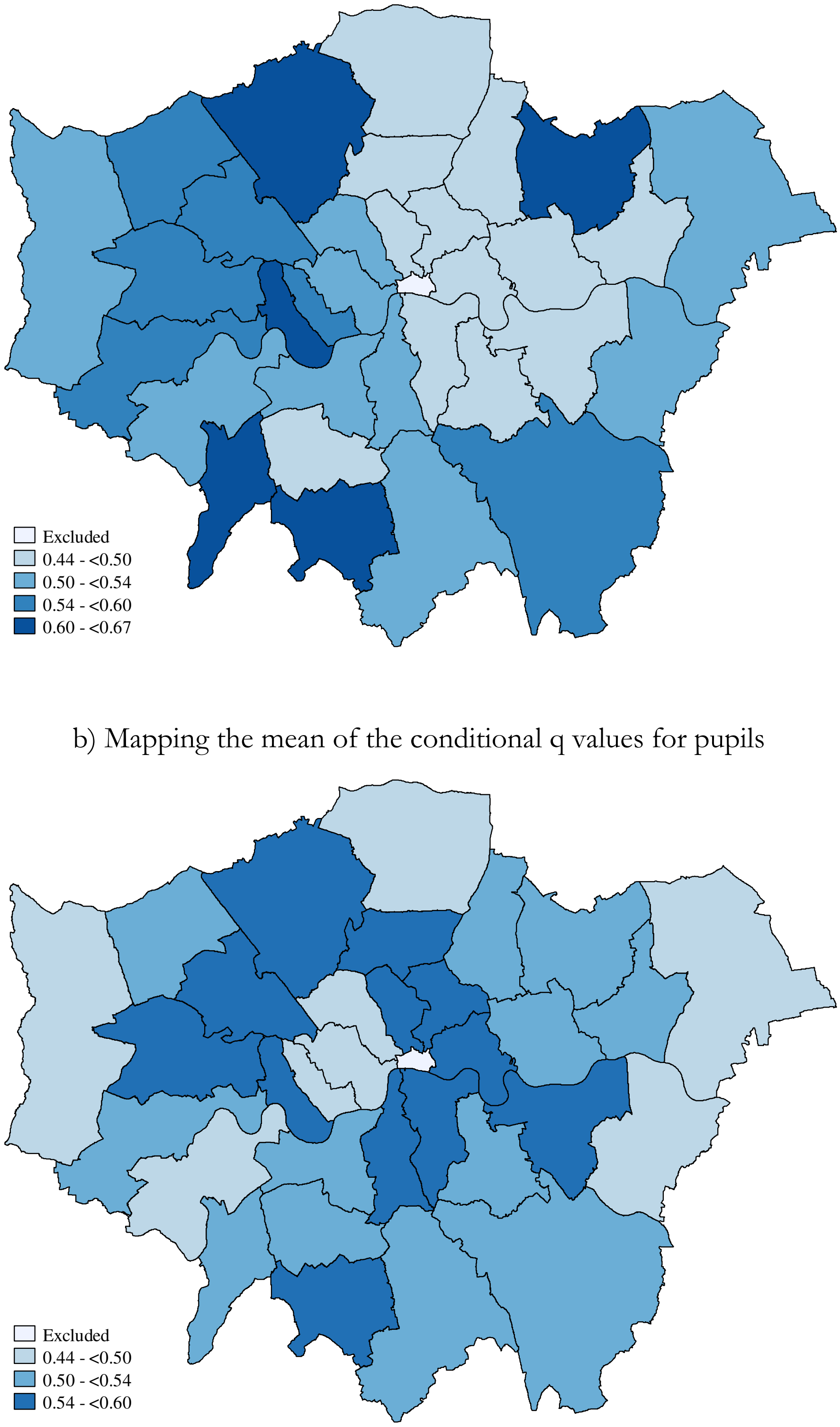


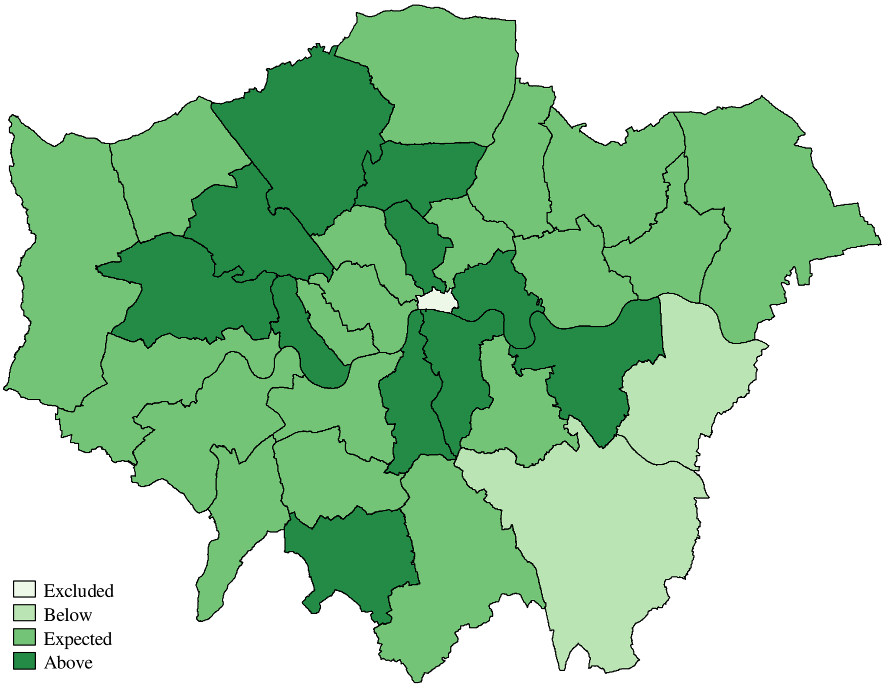
Figure 4: Mapping pupil performance across the Local Authorities of London

a) Mapping the mean of the marginal q values for pupils



b) Mapping the mean of the conditional q values for pupils

Figure 5: Highlighting Local Authorities with a mean q-value significantly above the overall pupil mean (overall mean not within the estimated 95% confidence interval for the Local Authority mean)


Excluded
Below
Expected
Above